EX 9.1.1: A manufacturing plant produces a certain type of brake pad used in big rig trucks.

A sample of 55 day-shift workers showed that the mean # of brake pads produced was 310 with a std dev of 20.

A sample of 60 night-shift workers showed that the mean # of pads produced was 325 with a std dev of 26.

Do these samples suggest that the average # brake pads produced by the two shifts differ??

(Use significance level $\alpha = 0.05$)

(a) State the appropriate null hypothesis H_0 & alternative hypothesis H_A.

Let $\mu_1 \equiv$ (Mean # of brake pads produced by day shift)

Let $\mu_2 \equiv$ (Mean # of brake pads produced by night shift)

$$\begin{array}{l} H_0: \ \mu_1 = \mu_2 \\ H_A: \ \mu_1 \neq \mu_2 \end{array} \implies \boxed{\begin{array}{l} H_0: \ \mu_1 - \mu_2 = 0 \\ H_A: \ \mu_1 - \mu_2 \neq 0 \end{array}}$$

(b) Compute the appropriate test statistic value for this hypothesis test.

$$z = \frac{(\overline{x} - \overline{y}) - \delta_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(310 - 325) - 0}{\sqrt{\frac{20^2}{55} + \frac{26^2}{60}}} \approx \boxed{\mathbf{-3.48372}}$$

(c) Compute the resulting P-value.

$$\text{P-value } \approx 2 \cdot [1 - \Phi(|z|)] = 2 \cdot [1 - \Phi(3.48)] \overset{LOOKUP}{\approx} 2 \cdot [1 - 0.99975] = \boxed{\mathbf{0.0005}}$$

(d) Make the appropriate decision.

Since P-value $= 0.0005 \leq 0.05 = \alpha$, $\boxed{\textbf{Reject } H_0 \textbf{ in favor of } H_A}$

The sample evidence is compelling enough to conclude that

it's plausible that the avg # of brake pads produced by the two shifts differ.

(e) Construct the approximate 95% CI for $\mu_1 - \mu_2$.

$$(\overline{x} - \overline{y}) \pm z^*_{\alpha/2} \cdot \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

Approximate 95% CI for $\mu_1 - \mu_2$:

$$(310 - 325) \pm \mathbf{1.96} \cdot \sqrt{\frac{20^2}{55} + \frac{26^2}{60}} \implies -15 \pm 8.4392 \implies (-15 - 8.4392,\ -15 + 8.4392) = \boxed{(\mathbf{-23.4392},\ \mathbf{-6.5608})}$$

Note that the negative signs in the CI indicate that the night shift produces more brake pads on average than the day shift.

(f) What's the minimum sample size needed to construct an approx 95% CI for estimating $\mu_1 - \mu_2$ to within 1 brake pad?

$w = 1$, $z^*_{\alpha/2} = 1.96$, Best guesses for population std dev's σ_1, σ_2 are: $s_1 = 20$, $s_2 = 26$

$$\therefore \quad n = \left\lceil \frac{4(z^*_{\alpha/2})^2(s_1^2 + s_2^2)}{w^2} \right\rceil = \left\lceil \left(\frac{(4)(1.96)^2(20^2 + 26^2)}{1} \right)^2 \right\rceil = \lceil 16534.2464 \rceil = \boxed{\mathbf{16535}}$$