LARGE-SAMPLE TESTS/CI'S FOR $\mu_1 - \mu_2$ [DEVORE 9.1]

- **LARGE-SAMPLE z-TEST FOR $\mu_1 - \mu_2$ (σ_1, σ_2 UNKNOWN):**

Populations:	Any Two Populations with σ_1, σ_2 unknown
Random Samples:	$\mathbf{X} := (X_1, X_2, \ldots, X_{n_1})$ from 1^{st} population ($n_1 > 40$) $\mathbf{Y} := (Y_1, Y_2, \ldots, Y_{n_2})$ from 2^{nd} population ($n_2 > 40$) Random Samples $\mathbf{X}$ & $\mathbf{Y}$ are independent of one another
Realized Samples:	$\mathbf{x} := (x_1, x_2, \ldots, x_{n_1})$ from 1^{st} population with mean $\overline{x}$ & std dev s_1 $\mathbf{y} := (y_1, y_2, \ldots, y_{n_2})$ from 2^{nd} population with mean $\overline{y}$ & std dev s_2
Test Statistic Value $W(\mathbf{x}, \mathbf{y}; \delta_0)$	$z = \dfrac{(\overline{x} - \overline{y}) - \delta_0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$

HYPOTHESIS TEST:	**P-VALUE DETERMINATION:**
$H_0: \ \mu_1 - \mu_2 = \delta_0$ vs. $H_A: \ \mu_1 - \mu_2 > \delta_0$	P-value $\approx 1 - \Phi(z)$
$H_0: \ \mu_1 - \mu_2 = \delta_0$ vs. $H_A: \ \mu_1 - \mu_2 < \delta_0$	P-value $\approx \Phi(z)$
$H_0: \ \mu_1 - \mu_2 = \delta_0$ vs. $H_A: \ \mu_1 - \mu_2 \neq \delta_0$	P-value $\approx 2 \cdot [1 - \Phi(\|z\|)]$

Decision Rule:
If P-value $\leq \alpha$ then reject H_0 in favor of H_A
If P-value $> \alpha$ then accept H_0 (i.e. fail to reject H_0)

- **LARGE-SAMPLE z-CI FOR $\mu_1 - \mu_2$ (σ_1, σ_2 UNKNOWN):**

Given <u>any</u> two populations with means μ_1 and μ_2.

Let $x_1, x_2, \ldots, x_{n_1}$ be a large sample ($n_1 > 40$) with mean $\overline{x}$ and std dev s_1 taken from the 1^{st} population.

Let $y_1, y_2, \ldots, y_{n_2}$ be a large sample ($n_2 > 40$) with mean $\overline{y}$ and std dev s_2 taken from the 2^{nd} population.

Then the $100(1-\alpha)\%$ **large-sample CI for** $\mu_1 - \mu_2$ is approximately

$$\left((\overline{x} - \overline{y}) - z^*_{\alpha/2} \cdot \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}, \quad (\overline{x} - \overline{y}) + z^*_{\alpha/2} \cdot \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \right)$$

— OR WRITTEN MORE COMPACTLY —

$$(\overline{x} - \overline{y}) \pm z^*_{\alpha/2} \cdot \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

If a half-width of w is desired for the $100(1-\alpha)\%$ CI yielding $(\overline{x} - \overline{y}) \pm w$,

then the minimum sample size n required to achieve this is:

$$n = \left\lceil \frac{4(z^*_{\alpha/2})^2(s_1^2 + s_2^2)}{w^2} \right\rceil,$$ where s_1, s_2 are "best guesses" for the population std dev's σ_1, σ_2

EX 9.1.1: A manufacturing plant produces a certain type of brake pad used in big rig trucks.

A sample of 55 day-shift workers showed that the mean # of brake pads produced was 310 with a std dev of 20.

A sample of 60 night-shift workers showed that the mean # of pads produced was 325 with a std dev of 26.

Do these samples suggest that the average # brake pads produced by the two shifts differ??

(Use significance level $\alpha = 0.05$)

(a) State the appropriate null hypothesis H_0 & alternative hypothesis H_A.

(b) Compute the appropriate test statistic value for this hypothesis test.

(c) Compute the resulting P-value.

(d) Make the appropriate decision.

(e) Construct the approximate 95% CI for $\mu_1 - \mu_2$.

(f) What's the minimum sample size needed to construct an approx 95% CI for estimating $\mu_1 - \mu_2$ to within 1 brake pad?