EX 9.2.1: A manufacturing plant produces a certain type of brake pad used in big rig trucks.

A sample of 10 day-shift workers showed that the mean # of brake pads produced was 52 with a std dev of 4.4.

A sample of 13 night-shift workers showed that the mean # of brake pads produced was 55 with a std dev of 4.6.

Assume that the two corresponding populations are normally distributed.

Do these samples suggest that the average # brake pads produced by the two shifts differ??

(Use significance level $\alpha = 0.01$)

(a) State the appropriate null hypothesis H_0 & alternative hypothesis H_A.

Let $\mu_1 \equiv$ (Mean # of brake pads produced by day shift)

Let $\mu_2 \equiv$ (Mean # of brake pads produced by night shift)

$$\begin{array}{l} H_0: \ \mu_1 = \mu_2 \\ H_A: \ \mu_1 \neq \mu_2 \end{array} \implies \boxed{\begin{array}{l} H_0: \ \mu_1 - \mu_2 = 0 \\ H_A: \ \mu_1 - \mu_2 \neq 0 \end{array}}$$

(b) Compute the appropriate test statistic value for this hypothesis test.

$$t = \frac{(\overline{x} - \overline{y}) - \delta_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(52 - 55) - 0}{\sqrt{\frac{4.4^2}{10} + \frac{4.6^2}{13}}} \approx \boxed{\mathbf{-1.58917}}$$

(c) Compute the resulting P-value.

First, determine the appropriate # degrees of freedom:

$$\nu^* = \left\lfloor \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{(s_1^2/n_1)^2}{n_1 - 1} + \frac{(s_2^2/n_2)^2}{n_2 - 1}} \right\rfloor = \left\lfloor \frac{\left(\frac{4.4^2}{10} + \frac{4.6^2}{13}\right)^2}{\frac{(4.4^2/10)^2}{10 - 1} + \frac{(4.6^2/13)^2}{13 - 1}} \right\rfloor = \left\lfloor \frac{12.69990286}{0.637236965} \right\rfloor = \lfloor 19.92963929 \rfloor = 19$$

P-value $= 2 \cdot [1 - \Phi_t(|t|; \nu^*)] = 2 \cdot [1 - \Phi_t(1.6; \nu = 19)] \overset{LOOKUP}{\approx} 2 \cdot [1 - 0.937] = \boxed{\mathbf{0.126}}$

(d) Make the appropriate decision.

Since P-value $= 0.126 > 0.01 = \alpha$, **Accept (or Fail to Reject)** H_0

There is not enough compelling evidence to conclude that the avg # of brake pads produced by the two shifts differ.

(e) Construct the 99% CI for $\mu_1 - \mu_2$.

$$(\overline{x} - \overline{y}) \pm t^*_{\nu^*, \alpha/2} \cdot \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

99% CI for $\mu_1 - \mu_2$: $\quad t^*_{\nu^*, \alpha/2} = t^*_{19, 0.005} \overset{LOOKUP}{\approx} 2.861$

$$(52 - 55) \pm \mathbf{2.861} \cdot \sqrt{\frac{4.4^2}{10} + \frac{4.6^2}{13}} \implies -3 \pm 5.4009 \implies (-3 - 5.4009, \ -3 + 5.4009) = \boxed{(\mathbf{-8.4009}, \ \mathbf{2.4009})}$$

A negative sign in the CI indicates that the night shift produces more brake pads on average than the day shift.

A positive sign in the CI indicates that the day shift produces more brake pads on average than the night shift.