SMALL-SAMPLE TESTS/CI'S FOR $\mu_1 - \mu_2$ [DEVORE 9.2]

- **SMALL-SAMPLE t-TEST FOR $\mu_1 - \mu_2$ (σ_1, σ_2 UNKNOWN):**

Populations:	Two Normal Populations with σ_1, σ_2 unknown
Realized Samples:	$\mathbf{x} := (x_1, x_2, \ldots, x_{n_1})$ with mean $\overline{x}$ and std dev s_1 $\mathbf{y} := (y_1, y_2, \ldots, y_{n_2})$ with mean $\overline{y}$ and std dev s_2 Samples $\mathbf{x}$ & $\mathbf{y}$ are independent of one another
Test Statistic Value $W(\mathbf{x}, \mathbf{y}; \delta_0)$	$t = \dfrac{(\overline{x} - \overline{y}) - \delta_0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$ $\quad \nu^* = \left\lfloor \dfrac{\left(\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}\right)^2}{\dfrac{(s_1^2/n_1)^2}{n_1 - 1} + \dfrac{(s_2^2/n_2)^2}{n_2 - 1}} \right\rfloor$

HYPOTHESIS TEST:	P-VALUE DETERMINATION:
$H_0: \mu_1 - \mu_2 = \delta_0$ vs. $H_A: \mu_1 - \mu_2 > \delta_0$	P-value $= 1 - \Phi_t(t; \nu^*)$
$H_0: \mu_1 - \mu_2 = \delta_0$ vs. $H_A: \mu_1 - \mu_2 < \delta_0$	P-value $= \Phi_t(t; \nu^*)$
$H_0: \mu_1 - \mu_2 = \delta_0$ vs. $H_A: \mu_1 - \mu_2 \neq \delta_0$	P-value $= 2 \cdot [1 - \Phi_t(\lvert t \rvert; \nu^*)]$

Decision Rule:
If P-value $\leq \alpha$ then reject H_0 in favor of H_A
If P-value $> \alpha$ then accept H_0 (i.e. fail to reject H_0)

- **SMALL-SAMPLE t-CI FOR $\mu_1 - \mu_2$ (σ_1, σ_2 UNKNOWN):**

Given two normal populations with means μ_1 and μ_2.

Let $x_1, x_2, \ldots, x_{n_1}$ be a sample with mean $\overline{x}$ and std dev s_1 taken from the 1^{st} population.

Let $y_1, y_2, \ldots, y_{n_2}$ be a sample with mean $\overline{y}$ and std dev s_2 taken from the 2^{nd} population.

Then the $100(1-\alpha)\%$ **small-sample CI for** $\mu_1 - \mu_2$ is

$$\left((\overline{x} - \overline{y}) - t^*_{\nu^*, \alpha/2} \cdot \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}, \quad (\overline{x} - \overline{y}) + t^*_{\nu^*, \alpha/2} \cdot \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \right)$$

— OR WRITTEN MORE COMPACTLY —

$$(\overline{x} - \overline{y}) \pm t^*_{\nu^*, \alpha/2} \cdot \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

where

$$\nu^* = \left\lfloor \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{(s_1^2/n_1)^2}{n_1 - 1} + \frac{(s_2^2/n_2)^2}{n_2 - 1}} \right\rfloor$$

EX 9.2.1: A manufacturing plant produces a certain type of brake pad used in big rig trucks.

A sample of 10 day-shift workers showed that the mean # of brake pads produced was 52 with a std dev of 4.4.

A sample of 13 night-shift workers showed that the mean # of brake pads produced was 55 with a std dev of 4.6.

Assume that the two corresponding populations are normally distributed.

Do these samples suggest that the average # brake pads produced by the two shifts differ??

(Use significance level $\alpha = 0.01$)

(a) State the appropriate null hypothesis H_0 & alternative hypothesis H_A.

(b) Compute the appropriate test statistic value for this hypothesis test.

(c) Compute the resulting P-value.

(d) Make the appropriate decision.

(e) Construct the 99% CI for $\mu_1 - \mu_2$.