EX 9.4.1: The manufacturer of a sore throat medication has developed a new formulation that it claims to be more effective.

To evaluate the new medication, a **test group** of 120 current users try the new formulation.

After a four-week trial, 108 users from the test group indicated the new formulation was more effective.

Simultaneously, a **control group** of 150 users were given the current formulation but told it was the new one.

After a four-week trial, 114 users from the control group indicated it was an improvement.

Assume this is a **randomized clinical trial**, which means there was no bias/dishonesty/cheating involved.

Does the data suggest that the new formulation is more effective at treating a sore throat??

(Use significance level $\alpha = 0.05$)

(a) State the appropriate null hypothesis H_0 & alternative hypothesis H_A.

Let $p_1 \equiv$ (Proportion of all users who indicate the new formulation is more effective)

Let $p_2 \equiv$ (Proportion of all users who indicate the current formulation is more effective)

$$\begin{array}{l} H_0: \; p_1 = p_2 \\ H_A: \; p_1 > p_2 \end{array} \implies \boxed{\begin{array}{l} H_0: \; p_1 - p_2 = 0 \\ H_A: \; p_1 - p_2 > 0 \end{array}}$$

(b) Compute the appropriate test statistic value for this hypothesis test.

$$\widehat{p}_1 := \frac{X}{n_1} \equiv \frac{\#\text{ Successes in test group}}{\text{Sample Size of test group}} = \frac{108}{120} \implies \widehat{q}_1 := 1 - \widehat{p}_1 = 1 - \frac{108}{120} = \frac{12}{120}$$

$$\widehat{p}_2 := \frac{Y}{n_2} \equiv \frac{\#\text{ Successes in control group}}{\text{Sample Size of control group}} = \frac{114}{150} \implies \widehat{q}_2 := 1 - \widehat{p}_2 = 1 - \frac{114}{150} = \frac{36}{150}$$

$$\widehat{p} := \frac{X+Y}{n_1+n_2} \equiv \frac{\text{Total }\#\text{ Successes in both groups}}{\text{Total Sample Size of both groups}} = \frac{108+114}{120+150} = \frac{222}{270} \implies \widehat{q} := 1 - \widehat{p} = 1 - \frac{222}{270} = \frac{48}{270}$$

Note that this inference method is safe to use since: $\min\{n_1\widehat{p}_1, n_1\widehat{q}_1, n_2\widehat{p}_2, n_2\widehat{q}_2\} = \min\{108, 12, 114, 36\} = 12 \geq 10$

$$\therefore \quad z = \frac{\widehat{p}_1 - \widehat{p}_2}{\sqrt{\widehat{p}\widehat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{(108/120) - (114/150)}{\sqrt{\left(\frac{222}{270}\right)\left(\frac{48}{270}\right)\left(\frac{1}{120} + \frac{1}{150}\right)}} \approx \boxed{\mathbf{2.98985}}$$

(c) Compute the resulting P-value.

$$\text{P-value} \approx 1 - \Phi(z) = 1 - \Phi(2.99) \overset{LOOKUP}{\approx} 1 - 0.99861 = \boxed{\mathbf{0.00139}}$$

(d) Make the appropriate decision.

Since P-value $\approx 0.00139 \leq 0.05 = \alpha$, $\boxed{\textbf{Reject } H_0 \textbf{ in favor of } H_A}$

There is enough compelling evidence to suggest that

the new formulation is plausibly more effective at treating a sore throat than the current one.

(e) Construct the approximate two-sided 90% large-sample CI for $p_1 - p_2$.

$$\text{Approximate 90\% CI for } p_1 - p_2: \qquad (\widehat{p}_1 - \widehat{p}_2) \pm z^*_{\alpha/2} \cdot \sqrt{\frac{\widehat{p}_1\widehat{q}_1}{n_1} + \frac{\widehat{p}_2\widehat{q}_2}{n_2}}$$

$$\left(\frac{108}{120} - \frac{114}{150}\right) \pm \mathbf{1.64} \cdot \sqrt{\frac{(108/120)(12)(120)}{120} + \frac{(114/150)(36/150)}{150}}$$

$$\implies 0.14 \pm 0.07272 \implies (0.14 - 0.07272,\; 0.14 + 0.07272) = \boxed{(\mathbf{0.06728},\; \mathbf{0.21272})}$$