Discrete Random Variables

Engineering Statistics
Section 3.1

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Random Variables & Their Supports (Definition)

Often each outcome of an experiment can be associated with a real number by means of a function or mapping:

Definition

(Random Variable)

Given an experiment with sample space Ω. Then:

$$X \text{ is a } \textbf{random variable (r.v.)} \text{ for the experiment} \iff X : \Omega \to \mathbb{R}$$

i.e. X is a function that maps each outcome in the sample space to a real #.

NOTATION: Random variables are denoted by capital letters: X, Y, Z, U, V, W
Similar random variables have subscripts: $X_1, X_2, X_3, X_4, \ldots$

Definition

(Support of a Random Variable)

The **support** of a random variable is the **set of meaningful values** for it.

NOTATION: The support of random variable X is denoted as $\text{Supp}(X)$.

Discrete & Continuous Random Variables (Definitions)

There are two types of random variables:

Definition

(Discrete Random Variable)

X is a **discrete random variable** $\iff$ Supp(X) is countable.

i.e. The meaningful values of X comprise a subset of integers $\mathbb{Z}$ or rationals $\mathbb{Q}$.

Definition

(Continuous Random Variable)

X is a **continuous random variable** $\iff$ Supp(X) is uncountable.

i.e. The meaningful values of X comprise an interval or union of intervals or $\mathbb{R}$.

NOTE: Continuous random variables will be explored in Chapter 4.

Discrete r.v.'s & Their Supports (Examples)

Experiment: Observe which seats in a 3-seat car are occupied (F) or not (A)
Sample Space: $\Omega = \{AAA, AAF, AFA, AFF, FAA, FAF, FFA, FFF\}$

Let random variables

$W \equiv$ If 3^{rd} seat in car is available (1 = Yes, 0 = No)
$X \equiv$ If car has any available seats (1 = Yes, 0 = No)
$Y \equiv$ Number of available seats in car
$Z \equiv$ Difference in # of available and occupied seats

Then:

$$
\begin{array}{llll}
W(AAA)=1 & X(AAA)=1 & Y(AAA)=3 & Z(AAA)=3-0 = 3 \\
W(AAF)=0 & X(AAF)=1 & Y(AAF)=2 & Z(AAF)=2-1 = 1 \\
W(AFA)=1 & X(AFA)=1 & Y(AFA)=2 & Z(AFA)=2-1 = 1 \\
W(AFF)=0 & X(AFF)=1 & Y(AFF)=1 & Z(AFF)=1-2 = -1 \\
W(FAA)=1 & X(FAA)=1 & Y(FAA)=2 & Z(FAA)=2-1 = 1 \\
W(FAF)=0 & X(FAF)=1 & Y(FAF)=1 & Z(FAF)=1-2 = -1 \\
W(FFA)=1 & X(FFA)=1 & Y(FFA)=1 & Z(FFA)=1-2 = -1 \\
W(FFF)=0 & X(FFF)=0 & Y(FFF)=0 & Z(FFF)=0-3 = -3
\end{array}
$$

Supp$(W) = \{0,1\} \implies$ Supp(W) is countable $\implies W$ is discrete
Supp$(X) = \{0,1\} \implies$ Supp(X) is countable $\implies X$ is discrete
Supp$(Y) = \{0,1,2,3\} \implies$ Supp(Y) is countable $\implies Y$ is discrete
Supp$(Z) = \{-3,-1,1,3\} \implies$ Supp(Z) is countable $\implies Z$ is discrete

Discrete r.v.'s & Probabilities (Examples)

Experiment: Observe which seats in a 3-seat car are occupied (F) or not (A)
Sample Space: $\Omega = \{AAA, AAF, AFA, AFF, FAA, FAF, FFA, FFF\}$

Let random variables

$W \equiv$ If 3^{rd} seat in car is available (1 = Yes, 0 = No)
$X \equiv$ If car has any available seats (1 = Yes, 0 = No)
$Y \equiv$ Number of available seats in car
$Z \equiv$ Difference in # of available and occupied seats

Then:

W(AAA) = 1	$X(AAA) = 1$	$Y(AAA) = 3$	$Z(AAA) = 3 - 0 = 3$
$W(AAF) = 0$	$X(AAF) = 1$	$Y(AAF) = 2$	$Z(AAF) = 2 - 1 = 1$
W(AFA) = 1	$X(AFA) = 1$	$Y(AFA) = 2$	$Z(AFA) = 2 - 1 = 1$
$W(AFF) = 0$	$X(AFF) = 1$	$Y(AFF) = 1$	$Z(AFF) = 1 - 2 = -1$
W(FAA) = 1	$X(FAA) = 1$	$Y(FAA) = 2$	$Z(FAA) = 2 - 1 = 1$
$W(FAF) = 0$	$X(FAF) = 1$	$Y(FAF) = 1$	$Z(FAF) = 1 - 2 = -1$
W(FFA) = 1	$X(FFA) = 1$	$Y(FFA) = 1$	$Z(FFA) = 1 - 2 = -1$
$W(FFF) = 0$	$X(FFF) = 0$	$Y(FFF) = 0$	$Z(FFF) = 0 - 3 = -3$

$$\begin{aligned}\mathbb{P}(3^{rd}\text{ seat is available}) &= \mathbb{P}(\mathbf{W = 1}) \\ &= \mathbb{P}(\omega \in \{AAA, AFA, FAA, FFA\}) \\ &= \frac{|\{AAA, AFA, FAA, FFA\}|}{|\Omega|} = \frac{4}{8} = \boxed{\frac{1}{2}}\end{aligned}$$

Discrete r.v.'s & Probabilities (Examples)

Experiment: Observe which seats in a 3-seat car are occupied (F) or not (A)
Sample Space: $\Omega = \{AAA, AAF, AFA, AFF, FAA, FAF, FFA, FFF\}$

Let random variables
$W \equiv$ If 3^{rd} seat in car is available (1 = Yes, 0 = No)
$X \equiv$ If car has any available seats (1 = Yes, 0 = No)
$Y \equiv$ Number of available seats in car
$Z \equiv$ Difference in # of available and occupied seats

Then:

$W(AAA) = 1$	$X(AAA) = 1$	$Y(AAA) = 3$	$Z(AAA) = 3 - 0 = 3$
W(AAF) = 0	$X(AAF) = 1$	$Y(AAF) = 2$	$Z(AAF) = 2 - 1 = 1$
$W(AFA) = 1$	$X(AFA) = 1$	$Y(AFA) = 2$	$Z(AFA) = 2 - 1 = 1$
W(AFF) = 0	$X(AFF) = 1$	$Y(AFF) = 1$	$Z(AFF) = 1 - 2 = -1$
$W(FAA) = 1$	$X(FAA) = 1$	$Y(FAA) = 2$	$Z(FAA) = 2 - 1 = 1$
W(FAF) = 0	$X(FAF) = 1$	$Y(FAF) = 1$	$Z(FAF) = 1 - 2 = -1$
$W(FFA) = 1$	$X(FFA) = 1$	$Y(FFA) = 1$	$Z(FFA) = 1 - 2 = -1$
W(FFF) = 0	$X(FFF) = 0$	$Y(FFF) = 0$	$Z(FFF) = 0 - 3 = -3$

$$\begin{aligned}\mathbb{P}(3^{rd} \text{ seat is occupied}) &= \mathbb{P}(\mathbf{W = 0}) \\ &= \mathbb{P}(\omega \in \{AAF, AFF, FAF, FFF\}) \\ &= \frac{|\{AAF, AFF, FAF, FFF\}|}{|\Omega|} = \frac{4}{8} = \boxed{\frac{1}{2}}\end{aligned}$$

Discrete r.v.'s & Probabilities (Examples)

Experiment: Observe which seats in a 3-seat car are occupied (F) or not (A)
Sample Space: $\Omega = \{AAA, AAF, AFA, AFF, FAA, FAF, FFA, FFF\}$

Let random variables
$W \equiv$ If 3^{rd} seat in car is available (1 = Yes, 0 = No)
$X \equiv$ If car has any available seats (1 = Yes, 0 = No)
$Y \equiv$ Number of available seats in car
$Z \equiv$ Difference in # of available and occupied seats

Then:

$$\begin{array}{llll}
W(AAA)=1 & X(AAA)=1 & Y(AAA)=3 & Z(AAA)=3-0 = 3\\
W(AAF)=0 & X(AAF)=1 & Y(AAF)=2 & Z(AAF)=2-1 = 1\\
W(AFA)=1 & X(AFA)=1 & Y(AFA)=2 & Z(AFA)=2-1 = 1\\
W(AFF)=0 & X(AFF)=1 & Y(AFF)=1 & Z(AFF)=1-2 = -1\\
W(FAA)=1 & X(FAA)=1 & Y(FAA)=2 & Z(FAA)=2-1 = 1\\
W(FAF)=0 & X(FAF)=1 & Y(FAF)=1 & Z(FAF)=1-2 = -1\\
W(FFA)=1 & X(FFA)=1 & Y(FFA)=1 & Z(FFA)=1-2 = -1\\
W(FFF)=0 & \mathbf{X(FFF) = 0} & Y(FFF)=0 & Z(FFF)=0-3 = -3
\end{array}$$

$$\begin{aligned}
\mathbb{P}(\text{No seats are available}) &= \mathbb{P}(\mathbf{X = 0})\\
&= \mathbb{P}(\omega \in \{FFF\})\\
&= \frac{|\{FFF\}|}{|\Omega|} = \boxed{\frac{1}{8}}
\end{aligned}$$

Discrete r.v.'s & Probabilities (Examples)

Experiment: Observe which seats in a 3-seat car are occupied (F) or not (A)
Sample Space: $\Omega = \{AAA, AAF, AFA, AFF, FAA, FAF, FFA, FFF\}$

Let random variables

$W \equiv$ If 3^{rd} seat in car is available (1 = Yes, 0 = No)
$X \equiv$ If car has any available seats (1 = Yes, 0 = No)
$Y \equiv$ Number of available seats in car
$Z \equiv$ Difference in # of available and occupied seats

Then:

$W(AAA) = 1$	$X(AAA) = 1$	$Y(AAA) = 3$	$Z(AAA) = 3 - 0 = 3$
$W(AAF) = 0$	$X(AAF) = 1$	$Y(AAF) = 2$	$Z(AAF) = 2 - 1 = 1$
$W(AFA) = 1$	$X(AFA) = 1$	$Y(AFA) = 2$	$Z(AFA) = 2 - 1 = 1$
$W(AFF) = 0$	$X(AFF) = 1$	$Y(AFF) = 1$	$Z(AFF) = 1 - 2 = -1$
$W(FAA) = 1$	$X(FAA) = 1$	$Y(FAA) = 2$	$Z(FAA) = 2 - 1 = 1$
$W(FAF) = 0$	$X(FAF) = 1$	$Y(FAF) = 1$	$Z(FAF) = 1 - 2 = -1$
$W(FFA) = 1$	$X(FFA) = 1$	$Y(FFA) = 1$	$Z(FFA) = 1 - 2 = -1$
$W(FFF) = 0$	$X(FFF) = 0$	**Y(FFF) = 0**	$Z(FFF) = 0 - 3 = -3$

$$\begin{aligned}\mathbb{P}(\text{No seats are available}) &= \mathbb{P}(\mathbf{Y = 0}) \\ &= \mathbb{P}(\omega \in \{FFF\}) \\ &= \frac{|\{FFF\}|}{|\Omega|} = \boxed{\frac{1}{8}}\end{aligned}$$

Discrete r.v.'s & Probabilities (Examples)

Experiment: Observe which seats in a 3-seat car are occupied (F) or not (A)
Sample Space: $\Omega = \{AAA, AAF, AFA, AFF, FAA, FAF, FFA, FFF\}$

Let random variables
$W \equiv$ If 3^{rd} seat in car is available (1 = Yes, 0 = No)
$X \equiv$ If car has any available seats (1 = Yes, 0 = No)
$Y \equiv$ Number of available seats in car
$Z \equiv$ Difference in # of available and occupied seats

Then:

$$\begin{array}{llll}
W(AAA)=1 & X(AAA)=1 & Y(AAA)=3 & Z(AAA)=3-0 = 3 \\
W(AAF)=0 & X(AAF)=1 & Y(AAF)=2 & Z(AAF)=2-1 = 1 \\
W(AFA)=1 & X(AFA)=1 & Y(AFA)=2 & Z(AFA)=2-1 = 1 \\
W(AFF)=0 & X(AFF)=1 & Y(AFF)=1 & Z(AFF)=1-2 = -1 \\
W(FAA)=1 & X(FAA)=1 & Y(FAA)=2 & Z(FAA)=2-1 = 1 \\
W(FAF)=0 & X(FAF)=1 & Y(FAF)=1 & Z(FAF)=1-2 = -1 \\
W(FFA)=1 & X(FFA)=1 & Y(FFA)=1 & Z(FFA)=1-2 = -1 \\
W(FFF)=0 & X(FFF)=0 & Y(FFF)=0 & \mathbf{Z(FFF)}=0-3 = \mathbf{-3}
\end{array}$$

$$\begin{aligned}
\mathbb{P}(\text{No seats are available}) &= \mathbb{P}(\mathbf{Z = -3}) \\
&= \mathbb{P}(\omega \in \{FFF\}) \\
&= \frac{|\{FFF\}|}{|\Omega|} = \boxed{\frac{1}{8}}
\end{aligned}$$

Discrete r.v.'s & Probabilities (Examples)

Experiment: Observe which seats in a 3-seat car are occupied (F) or not (A)
Sample Space: $\Omega = \{AAA, AAF, AFA, AFF, FAA, FAF, FFA, FFF\}$

Let random variables
$W \equiv$ If 3^{rd} seat in car is available (1 = Yes, 0 = No)
$X \equiv$ If car has any available seats (1 = Yes, 0 = No)
$Y \equiv$ Number of available seats in car
$Z \equiv$ Difference in # of available and occupied seats

Then:

$W(AAA) = 1$	$X(AAA) = 1$	$Y(AAA) = 3$	$Z(AAA) = 3 - 0 = 3$
$W(AAF) = 0$	$X(AAF) = 1$	$Y(AAF) = 2$	$Z(AAF) = 2 - 1 = 1$
$W(AFA) = 1$	$X(AFA) = 1$	$Y(AFA) = 2$	$Z(AFA) = 2 - 1 = 1$
$W(AFF) = 0$	$X(AFF) = 1$	**Y(AFF) = 1**	$Z(AFF) = 1 - 2 = -1$
$W(FAA) = 1$	$X(FAA) = 1$	$Y(FAA) = 2$	$Z(FAA) = 2 - 1 = 1$
$W(FAF) = 0$	$X(FAF) = 1$	**Y(FAF) = 1**	$Z(FAF) = 1 - 2 = -1$
$W(FFA) = 1$	$X(FFA) = 1$	**Y(FFA) = 1**	$Z(FFA) = 1 - 2 = -1$
$W(FFF) = 0$	$X(FFF) = 0$	$Y(FFF) = 0$	$Z(FFF) = 0 - 3 = -3$

$$\begin{aligned}
\mathbb{P}(\text{One seat is available}) &= \mathbb{P}(\mathbf{Y = 1}) \\
&= \mathbb{P}(\omega \in \{AFF, FAF, FFA\}) \\
&= \frac{|\{AFF, FAF, FFA\}|}{|\Omega|} = \boxed{\frac{3}{8}}
\end{aligned}$$

Discrete r.v.'s & Probabilities (Examples)

Experiment: Observe which seats in a 3-seat car are occupied (F) or not (A)
Sample Space: $\Omega = \{AAA, AAF, AFA, AFF, FAA, FAF, FFA, FFF\}$

Let random variables
$W \equiv$ If 3^{rd} seat in car is available (1 = Yes, 0 = No)
$X \equiv$ If car has any available seats (1 = Yes, 0 = No)
$Y \equiv$ Number of available seats in car
$Z \equiv$ Difference in # of available and occupied seats

Then:

$$
\begin{array}{llll}
W(AAA) = 1 & X(AAA) = 1 & Y(AAA) = 3 & Z(AAA) = 3 - 0 = 3 \\
W(AAF) = 0 & X(AAF) = 1 & Y(AAF) = 2 & Z(AAF) = 2 - 1 = 1 \\
W(AFA) = 1 & X(AFA) = 1 & Y(AFA) = 2 & Z(AFA) = 2 - 1 = 1 \\
W(AFF) = 0 & X(AFF) = 1 & Y(AFF) = 1 & \mathbf{Z(AFF)} = 1 - 2 = \mathbf{-1} \\
W(FAA) = 1 & X(FAA) = 1 & Y(FAA) = 2 & Z(FAA) = 2 - 1 = 1 \\
W(FAF) = 0 & X(FAF) = 1 & Y(FAF) = 1 & \mathbf{Z(FAF)} = 1 - 2 = \mathbf{-1} \\
W(FFA) = 1 & X(FFA) = 1 & Y(FFA) = 1 & \mathbf{Z(FFA)} = 1 - 2 = \mathbf{-1} \\
W(FFF) = 0 & X(FFF) = 0 & Y(FFF) = 0 & Z(FFF) = 0 - 3 = -3
\end{array}
$$

$$
\begin{aligned}
\mathbb{P}(\text{One seat is available}) &= \mathbb{P}(\mathbf{Z = -1}) \\
&= \mathbb{P}(\omega \in \{AFF, FAF, FFA\}) \\
&= \frac{|\{AFF, FAF, FFA\}|}{|\Omega|} = \boxed{\frac{3}{8}}
\end{aligned}
$$

Discrete r.v.'s & Probabilities (Examples)

Experiment: Observe which seats in a 3-seat car are occupied (F) or not (A)
Sample Space: $\Omega = \{AAA, AAF, AFA, AFF, FAA, FAF, FFA, FFF\}$

Let random variables
$W \equiv$ If 3^{rd} seat in car is available (1 = Yes, 0 = No)
$X \equiv$ If car has any available seats (1 = Yes, 0 = No)
$Y \equiv$ Number of available seats in car
$Z \equiv$ Difference in # of available and occupied seats

Then:

$W(AAA) = 1$	$X(AAA) = 1$	$Y(AAA) = 3$	$Z(AAA) = 3 - 0 = 3$
$W(AAF) = 0$	$X(AAF) = 1$	**Y(AAF) = 2**	$Z(AAF) = 2 - 1 = 1$
$W(AFA) = 1$	$X(AFA) = 1$	**Y(AFA) = 2**	$Z(AFA) = 2 - 1 = 1$
$W(AFF) = 0$	$X(AFF) = 1$	$Y(AFF) = 1$	$Z(AFF) = 1 - 2 = -1$
$W(FAA) = 1$	$X(FAA) = 1$	**Y(FAA) = 2**	$Z(FAA) = 2 - 1 = 1$
$W(FAF) = 0$	$X(FAF) = 1$	$Y(FAF) = 1$	$Z(FAF) = 1 - 2 = -1$
$W(FFA) = 1$	$X(FFA) = 1$	$Y(FFA) = 1$	$Z(FFA) = 1 - 2 = -1$
$W(FFF) = 0$	$X(FFF) = 0$	$Y(FFF) = 0$	$Z(FFF) = 0 - 3 = -3$

$$\begin{aligned}\mathbb{P}(\text{Two seats are available}) &= \mathbb{P}(\mathbf{Y = 2}) \\ &= \mathbb{P}(\omega \in \{AAF, AFA, FAA\}) \\ &= \frac{|\{AAF, AFA, FAA\}|}{|\Omega|} = \boxed{\frac{3}{8}}\end{aligned}$$

Discrete r.v.'s & Probabilities (Examples)

Experiment: Observe which seats in a 3-seat car are occupied (F) or not (A)
Sample Space: $\Omega = \{AAA, AAF, AFA, AFF, FAA, FAF, FFA, FFF\}$

Let random variables

$W \equiv$ If 3^{rd} seat in car is available (1 = Yes, 0 = No)
$X \equiv$ If car has any available seats (1 = Yes, 0 = No)
$Y \equiv$ Number of available seats in car
$Z \equiv$ Difference in # of available and occupied seats

Then:

$W(AAA) = 1$	$X(AAA) = 1$	$Y(AAA) = 3$	$Z(AAA) = 3 - 0 = 3$
$W(AAF) = 0$	$X(AAF) = 1$	$Y(AAF) = 2$	**$Z(AAF)$** $= 2 - 1 =$ **1**
$W(AFA) = 1$	$X(AFA) = 1$	$Y(AFA) = 2$	**$Z(AFA)$** $= 2 - 1 =$ **1**
$W(AFF) = 0$	$X(AFF) = 1$	$Y(AFF) = 1$	$Z(AFF) = 1 - 2 = -1$
$W(FAA) = 1$	$X(FAA) = 1$	$Y(FAA) = 2$	**$Z(FAA)$** $= 2 - 1 =$ **1**
$W(FAF) = 0$	$X(FAF) = 1$	$Y(FAF) = 1$	$Z(FAF) = 1 - 2 = -1$
$W(FFA) = 1$	$X(FFA) = 1$	$Y(FFA) = 1$	$Z(FFA) = 1 - 2 = -1$
$W(FFF) = 0$	$X(FFF) = 0$	$Y(FFF) = 0$	$Z(FFF) = 0 - 3 = -3$

$$\begin{aligned}\mathbb{P}(\text{Two seats are available}) &= \mathbb{P}(\mathbf{Z = 1}) \\ &= \mathbb{P}(\omega \in \{AAF, AFA, FAA\}) \\ &= \frac{|\{AAF, AFA, FAA\}|}{|\Omega|} = \boxed{\frac{3}{8}}\end{aligned}$$

Discrete r.v.'s & Probabilities (Examples)

Experiment: Observe which seats in a 3-seat car are occupied (F) or not (A)
Sample Space: $\Omega = \{AAA, AAF, AFA, AFF, FAA, FAF, FFA, FFF\}$

Let random variables
$W \equiv$ If 3^{rd} seat in car is available (1 = Yes, 0 = No)
$X \equiv$ If car has any available seats (1 = Yes, 0 = No)
$Y \equiv$ Number of available seats in car
$Z \equiv$ Difference in # of available and occupied seats

Then:

$W(AAA) = 1$	$X(AAA) = 1$	**Y(AAA) = 3**	$Z(AAA) = 3 - 0 = 3$
$W(AAF) = 0$	$X(AAF) = 1$	$Y(AAF) = 2$	$Z(AAF) = 2 - 1 = 1$
$W(AFA) = 1$	$X(AFA) = 1$	$Y(AFA) = 2$	$Z(AFA) = 2 - 1 = 1$
$W(AFF) = 0$	$X(AFF) = 1$	$Y(AFF) = 1$	$Z(AFF) = 1 - 2 = -1$
$W(FAA) = 1$	$X(FAA) = 1$	$Y(FAA) = 2$	$Z(FAA) = 2 - 1 = 1$
$W(FAF) = 0$	$X(FAF) = 1$	$Y(FAF) = 1$	$Z(FAF) = 1 - 2 = -1$
$W(FFA) = 1$	$X(FFA) = 1$	$Y(FFA) = 1$	$Z(FFA) = 1 - 2 = -1$
$W(FFF) = 0$	$X(FFF) = 0$	$Y(FFF) = 0$	$Z(FFF) = 0 - 3 = -3$

$$\begin{aligned}\mathbb{P}(\text{All seats are available}) &= \mathbb{P}(\mathbf{Y = 3}) \\ &= \mathbb{P}(\omega \in \{AAA\}) \\ &= \frac{|\{AAA\}|}{|\Omega|} = \boxed{\frac{1}{8}}\end{aligned}$$

Discrete r.v.'s & Probabilities (Examples)

Experiment: Observe which seats in a 3-seat car are occupied (F) or not (A)
Sample Space: $\Omega = \{AAA, AAF, AFA, AFF, FAA, FAF, FFA, FFF\}$

Let random variables
$W \equiv$ If 3^{rd} seat in car is available (1 = Yes, 0 = No)
$X \equiv$ If car has any available seats (1 = Yes, 0 = No)
$Y \equiv$ Number of available seats in car
$Z \equiv$ Difference in # of available and occupied seats

Then:

$$
\begin{array}{llll}
W(AAA)=1 & X(AAA)=1 & Y(AAA)=3 & \mathbf{Z(AAA)} = 3-0 = \mathbf{3} \\
W(AAF)=0 & X(AAF)=1 & Y(AAF)=2 & Z(AAF) = 2-1 = 1 \\
W(AFA)=1 & X(AFA)=1 & Y(AFA)=2 & Z(AFA) = 2-1 = 1 \\
W(AFF)=0 & X(AFF)=1 & Y(AFF)=1 & Z(AFF) = 1-2 = -1 \\
W(FAA)=1 & X(FAA)=1 & Y(FAA)=2 & Z(FAA) = 2-1 = 1 \\
W(FAF)=0 & X(FAF)=1 & Y(FAF)=1 & Z(FAF) = 1-2 = -1 \\
W(FFA)=1 & X(FFA)=1 & Y(FFA)=1 & Z(FFA) = 1-2 = -1 \\
W(FFF)=0 & X(FFF)=0 & Y(FFF)=0 & Z(FFF) = 0-3 = -3
\end{array}
$$

$$
\begin{aligned}
\mathbb{P}(\text{All seats are available}) &= \mathbb{P}(\mathbf{Z = 3}) \\
&= \mathbb{P}(\omega \in \{AAA\}) \\
&= \frac{|\{AAA\}|}{|\Omega|} = \boxed{\frac{1}{8}}
\end{aligned}
$$

Discrete r.v.'s & Impossible Events (Example)

Experiment: Observe which seats in a 3-seat car are occupied (F) or not (A)
Sample Space: $\Omega = \{AAA, AAF, AFA, AFF, FAA, FAF, FFA, FFF\}$

Let random variables

$W \equiv$ If 3^{rd} seat in car is available (1 = Yes, 0 = No)
$X \equiv$ If car has any available seats (1 = Yes, 0 = No)
$Y \equiv$ Number of available seats in car
$Z \equiv$ Difference in # of available and occupied seats

Then:

$$\begin{array}{llll}
W(AAA)=1 & X(AAA)=1 & Y(AAA)=3 & Z(AAA)=3-0 = 3 \\
W(AAF)=0 & X(AAF)=1 & Y(AAF)=2 & Z(AAF)=2-1 = 1 \\
W(AFA)=1 & X(AFA)=1 & Y(AFA)=2 & Z(AFA)=2-1 = 1 \\
W(AFF)=0 & X(AFF)=1 & Y(AFF)=1 & Z(AFF)=1-2 = -1 \\
W(FAA)=1 & X(FAA)=1 & Y(FAA)=2 & Z(FAA)=2-1 = 1 \\
W(FAF)=0 & X(FAF)=1 & Y(FAF)=1 & Z(FAF)=1-2 = -1 \\
W(FFA)=1 & X(FFA)=1 & Y(FFA)=1 & Z(FFA)=1-2 = -1 \\
W(FFF)=0 & X(FFF)=0 & Y(FFF)=0 & Z(FFF)=0-3 = -3
\end{array}$$

Of course, an **impossible event** always has a probability of **zero**.
This occurs with a random variable when a desired value is **not** in its **support**:

$\mathbb{P}(\text{Four seats are available}) = \mathbb{P}(Y=4) = \boxed{0}$

This is an impossible event since the value $4 \notin \text{Supp}(Y)$.

Discrete r.v.'s & Probabilities (Be Careful!!)

Experiment: Observe which seats in a 3-seat car are occupied (F) or not (A)
Sample Space: $\Omega = \{AAA, AAF, AFA, AFF, FAA, FAF, FFA, FFF\}$

Let random variables
$W \equiv$ If 3^{rd} seat in car is available (1 = Yes, 0 = No)
$X \equiv$ If car has any available seats (1 = Yes, 0 = No)
$Y \equiv$ Number of available seats in car
$Z \equiv$ Difference in # of available and occupied seats

Then:

W(AAA) = 1	$X(AAA) = 1$	$Y(AAA) = 3$	$Z(AAA) = 3 - 0$	$=$	3
W(AAF) = 0	$X(AAF) = 1$	$Y(AAF) = 2$	$Z(AAF) = 2 - 1$	$=$	1
W(AFA) = 1	$X(AFA) = 1$	$Y(AFA) = 2$	$Z(AFA) = 2 - 1$	$=$	1
W(AFF) = 0	$X(AFF) = 1$	$Y(AFF) = 1$	$Z(AFF) = 1 - 2$	$=$	-1
W(FAA) = 1	$X(FAA) = 1$	$Y(FAA) = 2$	$Z(FAA) = 2 - 1$	$=$	1
W(FAF) = 0	$X(FAF) = 1$	$Y(FAF) = 1$	$Z(FAF) = 1 - 2$	$=$	-1
W(FFA) = 1	$X(FFA) = 1$	$Y(FFA) = 1$	$Z(FFA) = 1 - 2$	$=$	-1
W(FFF) = 0	$X(FFF) = 0$	$Y(FFF) = 0$	$Z(FFF) = 0 - 3$	$=$	-3

If a r.v. has value(s) that comprise not only the event in question but also other outcomes, then that particular r.v. is useless w.r.t the event in question!
In such a situation, pick a random variable that properly models said event.
$\mathbb{P}(1^{st}$ seat is available$) = \mathbb{P}(\omega \in \{AAA, AAF, AFA, AFF\}) = \mathbb{P}(W = ?)$

Discrete r.v.'s & Probabilities (Be Careful!!)

Experiment: Observe which seats in a 3-seat car are occupied (F) or not (A)
Sample Space: $\Omega = \{AAA, AAF, AFA, AFF, FAA, FAF, FFA, FFF\}$

Let random variables
$W \equiv$ If 3^{rd} seat in car is available (1 = Yes, 0 = No)
$X \equiv$ If car has any available seats (1 = Yes, 0 = No)
$Y \equiv$ Number of available seats in car
$Z \equiv$ Difference in # of available and occupied seats

Then:

$W(AAA) = 1$	**X(AAA) = 1**	$Y(AAA) = 3$	$Z(AAA) = 3 - 0$	$=$	3
$W(AAF) = 0$	**X(AAF) = 1**	$Y(AAF) = 2$	$Z(AAF) = 2 - 1$	$=$	1
$W(AFA) = 1$	**X(AFA) = 1**	$Y(AFA) = 2$	$Z(AFA) = 2 - 1$	$=$	1
$W(AFF) = 0$	**X(AFF) = 1**	$Y(AFF) = 1$	$Z(AFF) = 1 - 2$	$=$	-1
$W(FAA) = 1$	**X(FAA) = 1**	$Y(FAA) = 2$	$Z(FAA) = 2 - 1$	$=$	1
$W(FAF) = 0$	**X(FAF) = 1**	$Y(FAF) = 1$	$Z(FAF) = 1 - 2$	$=$	-1
$W(FFA) = 1$	**X(FFA) = 1**	$Y(FFA) = 1$	$Z(FFA) = 1 - 2$	$=$	-1
$W(FFF) = 0$	$X(FFF) = 0$	$Y(FFF) = 0$	$Z(FFF) = 0 - 3$	$=$	-3

If a r.v. has value(s) that comprise not only the event in question but also other outcomes, then that particular r.v. is useless w.r.t the event in question!
In such a situation, pick a random variable that properly models said event.
$\mathbb{P}(1^{st}$ seat is available$) = \mathbb{P}(\omega \in \{AAA, AAF, AFA, AFF\}) = \mathbb{P}(X = ?)$

Discrete r.v.'s & Probabilities (Be Careful!!)

Experiment: Observe which seats in a 3-seat car are occupied (F) or not (A)
Sample Space: $\Omega = \{AAA, AAF, AFA, AFF, FAA, FAF, FFA, FFF\}$

Let random variables
- $W \equiv$ If 3^{rd} seat in car is available (1 = Yes, 0 = No)
- $X \equiv$ If car has any available seats (1 = Yes, 0 = No)
- $Y \equiv$ Number of available seats in car
- $Z \equiv$ Difference in # of available and occupied seats

Then:

$W(AAA) = 1$	$X(AAA) = 1$	**Y(AAA) = 3**	$Z(AAA) = 3 - 0 = 3$
$W(AAF) = 0$	$X(AAF) = 1$	**Y(AAF) = 2**	$Z(AAF) = 2 - 1 = 1$
$W(AFA) = 1$	$X(AFA) = 1$	**Y(AFA) = 2**	$Z(AFA) = 2 - 1 = 1$
$W(AFF) = 0$	$X(AFF) = 1$	**Y(AFF) = 1**	$Z(AFF) = 1 - 2 = -1$
$W(FAA) = 1$	$X(FAA) = 1$	**Y(FAA) = 2**	$Z(FAA) = 2 - 1 = 1$
$W(FAF) = 0$	$X(FAF) = 1$	**Y(FAF) = 1**	$Z(FAF) = 1 - 2 = -1$
$W(FFA) = 1$	$X(FFA) = 1$	**Y(FFA) = 1**	$Z(FFA) = 1 - 2 = -1$
$W(FFF) = 0$	$X(FFF) = 0$	$Y(FFF) = 0$	$Z(FFF) = 0 - 3 = -3$

If a r.v. has value(s) that comprise not only the event in question but also other outcomes, then that particular r.v. is useless w.r.t the event in question!
In such a situation, pick a random variable that properly models said event.

$\mathbb{P}(1^{st}$ seat is available$) = \mathbb{P}(\omega \in \{AAA, AAF, AFA, AFF\}) = \mathbb{P}(Y = ?)$

Discrete r.v.'s & Probabilities (Be Careful!!)

Experiment: Observe which seats in a 3-seat car are occupied (F) or not (A)
Sample Space: $\Omega = \{AAA, AAF, AFA, AFF, FAA, FAF, FFA, FFF\}$

Let random variables
$W \equiv$ If 3^{rd} seat in car is available (1 = Yes, 0 = No)
$X \equiv$ If car has any available seats (1 = Yes, 0 = No)
$Y \equiv$ Number of available seats in car
$Z \equiv$ Difference in # of available and occupied seats

Then:

$W(AAA) = 1$	$X(AAA) = 1$	$Y(AAA) = 3$	**Z(AAA)** $= 3 - 0$	$=$ **3**
$W(AAF) = 0$	$X(AAF) = 1$	$Y(AAF) = 2$	**Z(AAF)** $= 2 - 1$	$=$ **1**
$W(AFA) = 1$	$X(AFA) = 1$	$Y(AFA) = 2$	**Z(AFA)** $= 2 - 1$	$=$ **1**
$W(AFF) = 0$	$X(AFF) = 1$	$Y(AFF) = 1$	**Z(AFF)** $= 1 - 2$	$=$ **-1**
$W(FAA) = 1$	$X(FAA) = 1$	$Y(FAA) = 2$	**Z(FAA)** $= 2 - 1$	$=$ **1**
$W(FAF) = 0$	$X(FAF) = 1$	$Y(FAF) = 1$	**Z(FAF)** $= 1 - 2$	$=$ **-1**
$W(FFA) = 1$	$X(FFA) = 1$	$Y(FFA) = 1$	**Z(FFA)** $= 1 - 2$	$=$ **-1**
$W(FFF) = 0$	$X(FFF) = 0$	$Y(FFF) = 0$	$Z(FFF) = 0 - 3$	$= -3$

If a r.v. has value(s) that comprise not only the event in question but also other outcomes, then that particular r.v. is useless w.r.t the event in question!
In such a situation, pick a random variable that properly models said event.

$\mathbb{P}(1^{st}$ seat is available$) = \mathbb{P}(\omega \in \{AAA, AAF, AFA, AFF\}) = \mathbb{P}(Z = ?)$

Discrete r.v.'s & Probabilities (Be Careful!!)

Experiment: Observe which seats in a 3-seat car are occupied (F) or not (A)
Sample Space: $\Omega = \{AAA, AAF, AFA, AFF, FAA, FAF, FFA, FFF\}$

Let random variables
$W \equiv$ If 3^{rd} seat in car is available (1 = Yes, 0 = No)
$X \equiv$ If car has any available seats (1 = Yes, 0 = No)
$Y \equiv$ Number of available seats in car
$Z \equiv$ Difference in # of available and occupied seats

Then:

$$\begin{array}{llll} W(AAA)=1 & X(AAA)=1 & Y(AAA)=3 & Z(AAA)=3-0 = 3 \\ W(AAF)=0 & X(AAF)=1 & Y(AAF)=2 & Z(AAF)=2-1 = 1 \\ W(AFA)=1 & X(AFA)=1 & Y(AFA)=2 & Z(AFA)=2-1 = 1 \\ W(AFF)=0 & X(AFF)=1 & Y(AFF)=1 & Z(AFF)=1-2 = -1 \\ W(FAA)=1 & X(FAA)=1 & Y(FAA)=2 & Z(FAA)=2-1 = 1 \\ W(FAF)=0 & X(FAF)=1 & Y(FAF)=1 & Z(FAF)=1-2 = -1 \\ W(FFA)=1 & X(FFA)=1 & Y(FFA)=1 & Z(FFA)=1-2 = -1 \\ W(FFF)=0 & X(FFF)=0 & Y(FFF)=0 & Z(FFF)=0-3 = -3 \end{array}$$

If a r.v. has value(s) that comprise not only the event in question but also other outcomes, then that particular r.v. is useless w.r.t the event in question!
In such a situation, pick a random variable that properly models said event.
$\mathbb{P}(1^{st}$ seat is available$) = \mathbb{P}(\omega \in \{AAA, AAF, AFA, AFF\}) = \mathbb{P}(V = 1)$
where random variable $V \equiv$ If 1^{st} seat in car is available (1 = Yes, 0 = No)

Textbook Logistics for Section 3.1

- Difference(s) in Terminology:

TEXTBOOK TERMINOLOGY	SLIDES/OUTLINE TERMINOLOGY
Null Event $\emptyset$	Empty Set $\emptyset$
Number of Outcomes in E	Measure of E

- Difference(s) in Notation:

CONCEPT	TEXTBOOK NOTATION	SLIDES/OUTLINE NOTATION
Sample Space	$\mathcal{S}$	Ω
Probability of Event	$P(A)$	$\mathbb{P}(A)$
Measure of Event	$N(A)$	$\|A\|$
Support of a r.v.	"All possible values of X"	Supp(X)
Support of a r.v.	D	Supp(X)

Textbook Logistics for Section 3.1

- Skip Definition of a **Bernoulli random variable** (pg 97)
  - Bernoulli random variables will implicity occur in this section, but we won't identify them as Bernoulli random variables.
  - Bernoulli random variables (and their interpretations) will be explicitly covered in Section 3.4
- Skip EXAMPLE 3.5 (pg 97)
  - EXAMPLE 3.5 involves a continuous random variable.
  - Continuous random variables will be covered in Chapter 4.

Fin.