Bernoulli & Binomial Distributions

Engineering Statistics
Section 3.4

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TTU

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PART I:

BERNOULLI DISTRIBUTION

Jakob Bernoulli (1654-1705)

Bernoulli Random Variables (Applications)

Bernoulli random variables are used to model the result of a **Bernoulli Trial**:

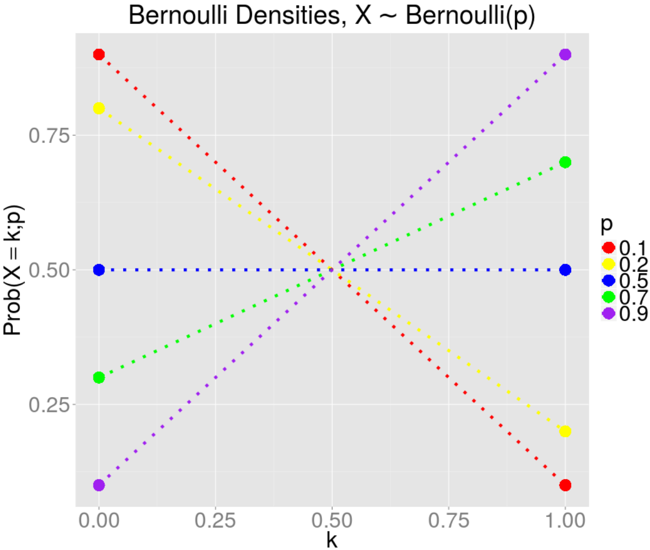
- Any random experiment where all possible outcomes are considered either a "Success" or "Failure" (but not both):
  - Flip a coin. (Success $\equiv$ "Heads", Failure $\equiv$ "Tails")
  - Flip a coin. (Success $\equiv$ "Tails", Failure $\equiv$ "Heads")
  - Roll a six-sided die. (Success $\equiv$ "6", Failure $\equiv$ "1,2,3,4 or 5")
  - Roll a six-sided die. (Success $\equiv$ "5 or 6", Failure $\equiv$ "1,2,3 or 4")
  - Roll a six-sided die. (Success $\equiv$ "odd #", Failure $\equiv$ "even #")
  - Shake a mixed bag of almonds & cashews until a single piece falls out. (Success $\equiv$ almond, Failure $\equiv$ cashew)
  - Randomly select a person in a large busy conference. (Success $\equiv$ person wearing a hat, Failure $\equiv$ person not wearing a hat)
  - Randomly select a person in the 'Treatment' group of a medical trial. (Success $\equiv$ treatment was effective, Failure $\equiv$ treatment was not effective)
- Any "Yes/No" question regarding some uncertain future event:
  - Will you vote 'yes' for the city proposal? (Success $\equiv$ "Yes", Failure $\equiv$ "No")
  - Is the next built widget defective? (Success $\equiv$ "No", Failure $\equiv$ "Yes")
  - Did the website get ≥ 1000 views today? (Success $\equiv$ "Yes", Failure $\equiv$ "No")
  - Is the newborn kitten female? (Success $\equiv$ "Yes", Failure $\equiv$ "No")

NOTE: "Success" & "Failure" are labels – do not interpret them literally.

Bernoulli Random Variables (Summary)

Proposition

Notation	$X \sim$ *Bernoulli*(p), $0 < p < 1$, $q := 1 - p$
Parameter(s)	$p \equiv \mathbb{P}($*Bernoulli Trial is a Success*$)$ $q \equiv \mathbb{P}($*Bernoulli Trial is a Failure*$)$
Support	*Supp*$(X) = \{0, 1\}$
Density (*pmf*)	$p_X(k;p) := p^k q^{1-k} = p^k (1-p)^{1-k}$
Mean	$\mathbb{E}[X] = p$
Variance	$\mathbb{V}[X] = pq = p(1-p)$
Model(s)	*Result of One Bernoulli Trial:* $1 \equiv$ *Success*, $0 \equiv$ *Failure*
Assumption(s)	*1. Random process has its sample space partitioned into Successes and Failures*



Remember, the only meaningful values of k for Bernoulli r.v.'s are 0 and 1.

Verification that Bernoulli pmf truly is a valid pmf

It's not immediately obvious that $p_X(k;p) = p^k q^{1-k}$ is a pmf, so let's prove it:

- Non-negativity on its support:

  Let $k \in \text{Supp}(X) = \{0,1\}$ and $0 < p < 1 \implies q = 1 - p$.
  Then $(1-k) \in \{0,1\}, 0 < q < 1 \implies p^k > 0, q^{1-k} > 0 \implies p_X(k;p) > 0$

- Universal Summation of Unity:

$$\sum_{k \in \text{Supp}(X)} p_X(k;p) = \sum_{k=0}^{1} p^k q^{1-k} \overset{(*)}{=} p^0 q^{1-0} + p^1 q^{1-1} = q + p = (1-p) + p = 1$$

$(*)$ Since the summation is **finite** with only two terms, simply expand it:

$$\text{e.g. } \sum_{k=0}^{3} a_k = a_0 + a_1 + a_2 + a_3$$

$$\text{e.g. } \sum_{k=0}^{3} k^2 = (0)^2 + (1)^2 + (2)^2 + (3)^2 = 0 + 1 + 4 + 9 = 14$$

Mean of Bernoulli(p) random variable (Proof)

Let random variable $X \sim$ Bernoulli(p), where $0 < p < 1$ and $q := 1 - p$. Then:

$$\begin{aligned} \mathbb{E}[X] &= \sum_{k \in \text{Supp}(X)} k \cdot p_X(k;p) = \sum_{k=0}^{1} k \cdot p^k q^{1-k} \overset{(*)}{=} (0) \cdot p^0 q^{1-0} + (1) \cdot p^1 q^{1-1} \\ &= 0 + pq^0 = 0 + p \cdot 1 = p \end{aligned}$$

$\therefore \quad \mathbb{E}[X] = p$ QED

$(*)$ Since the summation is **finite** with only two terms, simply expand it:

$$\text{e.g.} \quad \sum_{k=0}^{3} a_k = a_0 + a_1 + a_2 + a_3$$

$$\text{e.g.} \quad \sum_{k=0}^{3} k^2 = (0)^2 + (1)^2 + (2)^2 + (3)^2 = 0 + 1 + 4 + 9 = 14$$

Variance of Bernoulli(p) random variable (Proof)

Let random variable $X \sim$ Bernoulli(p), where $0 < p < 1$ and $q := 1 - p$. Then:

$$\begin{aligned} \mathbb{E}[X^2] &= \sum_{k \in \text{Supp}(X)} k^2 \cdot p_X(k;p) = \sum_{k=0}^{1} k^2 \cdot p^k q^{1-k} \overset{(*)}{=} (0)^2 \cdot p^0 q^{1-0} + (1)^2 \cdot p^1 q^{1-1} \\ &= 0 + 1 \cdot pq^0 = 0 + p = p \end{aligned}$$

$\therefore \quad \mathbb{V}[X] = \mathbb{E}[X^2] - (\mathbb{E}[X])^2 = p - (p)^2 = p - p^2 = p(1-p) = pq$ QED

$(*)$ Since the summation is **finite** with only two terms, simply expand it:

e.g. $\displaystyle\sum_{k=0}^{3} a_k = a_0 + a_1 + a_2 + a_3$

e.g. $\displaystyle\sum_{k=0}^{3} k^2 = (0)^2 + (1)^2 + (2)^2 + (3)^2 = 0 + 1 + 4 + 9 = 14$

PART II:

BINOMIAL DISTRIBUTION

Binomial Random Variables (Applications)

Binomial r. v.'s model the **# successes** of n **independent** Bernoulli Trials:

- Flip n coins, then count # Heads. (Success $\equiv$ "Heads", Failure $\equiv$ "Tails")
- Flip n coins, then count # Tails. (Success $\equiv$ "Tails", Failure $\equiv$ "Heads")

- Roll n dice, then count # 6's. (Success $\equiv$ "6", Failure $\equiv$ "1,2,3,4 or 5")
- Roll n dice, then count # 5's and 6's. (Success $\equiv$ "5 or 6")
- Roll n dice, then count # odd numbers. (Success $\equiv$ "odd #")

- Shake a mixed bag of almonds & cashews until n pieces fall out. Then count how many pieces are almonds. (Success $\equiv$ "almond piece")
- Randomly select n people in a large busy conference. Then count how many are wearing a hat. (Success $\equiv$ person wears hat)
- Randomly select n people in the 'Treatment' group of a medical trial. Then count how many had successful treatment.

- Randomly select n people, count how many will vote 'yes' for resolution.
- Count how many of the next n built widgets are **not** defective.
- Randomly select n websites, count how many got ≥ 1000 views today
- Randomly select n newborn kittens, count how many are female.

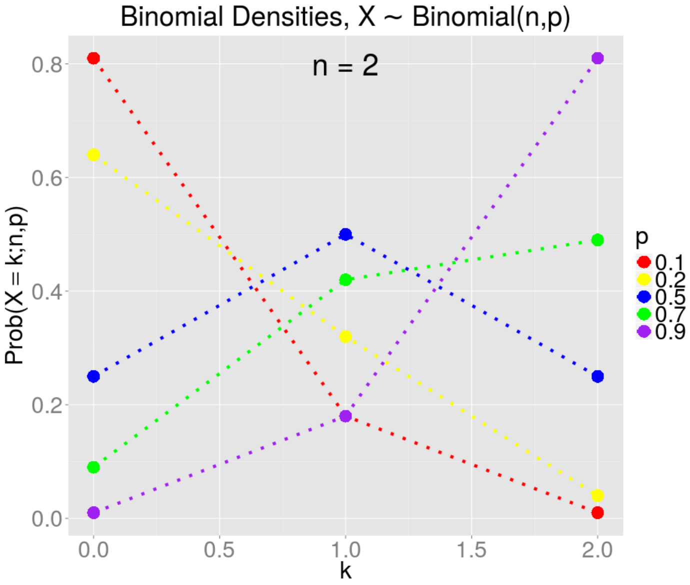
NOTE: "Success" & "Failure" are labels – do not interpret them literally.

Binomial Random Variables (Summary)

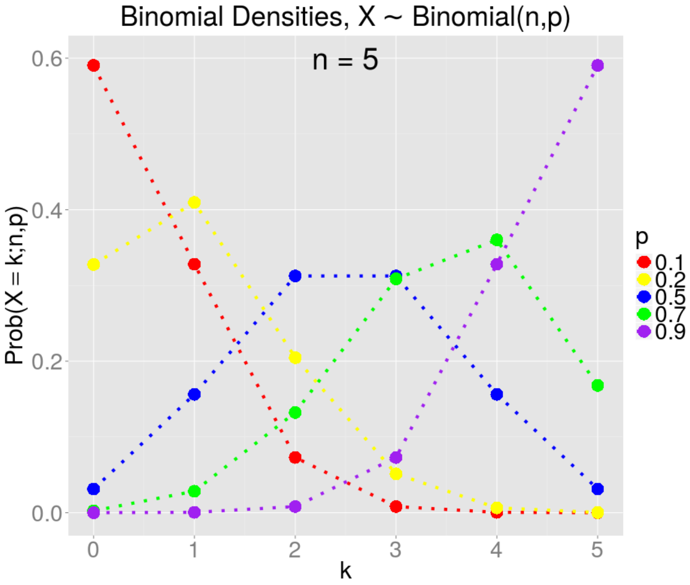
Proposition

Notation	$X \sim$ *Binomial*$(n, p), \quad n \geq 1, \quad 0 < p < 1, \quad q := 1 - p$
Parameter(s)	$p \equiv \mathbb{P}($*Bernoulli Trial is a Success*$)$ $q \equiv \mathbb{P}($*Bernoulli Trial is a Failure*$)$
Support	*Supp*$(X) = \{0, 1, 2, \cdots, n-2, n-1, n\}$
Density (*pmf*)	$p_X(k; n, p) := \binom{n}{k} p^k q^{n-k} = \binom{n}{k} p^k (1-p)^{n-k}$
Mean	$\mathbb{E}[X] = np$
Variance	$\mathbb{V}[X] = npq = np(1-p)$
Model(s)	*# Successes of* n *Bernoulli Trials*
Assumption(s)	*1. Random process comprises of* n *trials.* *2. Trials are all identical & independent.* *3. Random process has its sample space partitioned into Successes and Failures*

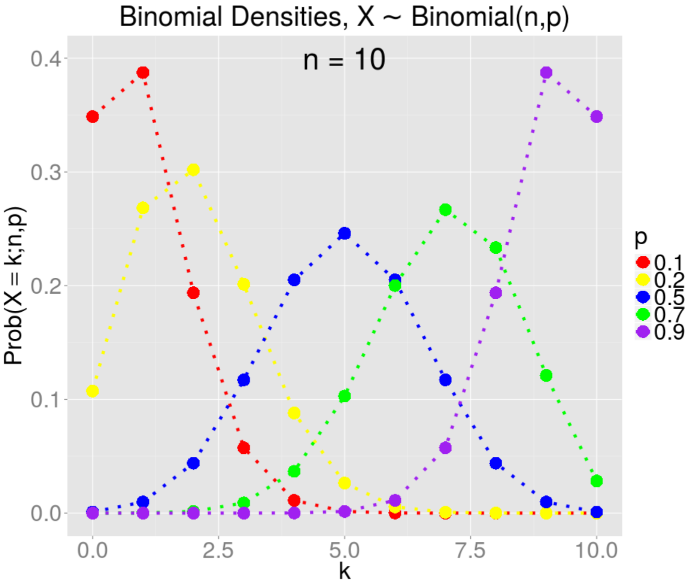
Binomial Density Plots (pmf's) for Sample Size $n = 2$



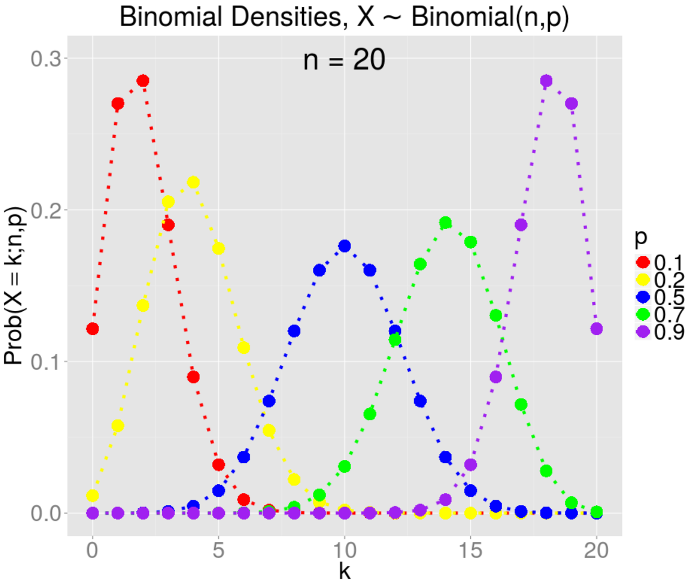
Binomial Density Plots (pmf's) for Sample Size $n = 5$



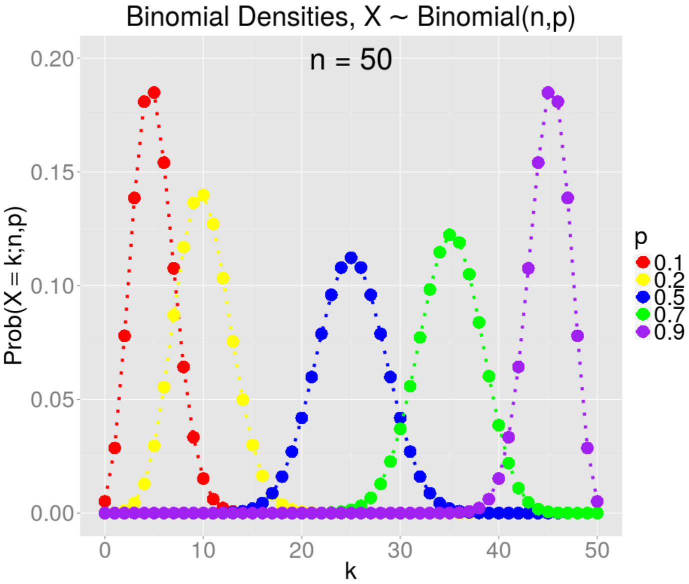
Binomial Density Plots (pmf's) for Sample Size $n = 10$



Binomial Density Plots (pmf's) for Sample Size $n = 20$



Binomial Density Plots (pmf's) for Sample Size $n = 50$



Verification that Binomial pmf truly is a valid pmf

It's not obvious that $p_X(k;n,p) = \binom{n}{k} p^k q^{n-k}$ is a pmf, so let's prove it:

- Non-negativity on its support:

  Let $n \geq 1,\ k \in \text{Supp}(X) = \{0, 1, 2, \cdots, n\}$ and $0 < p < 1 \implies q = 1 - p$.
  Then $(n-k) \geq 0,\ \ 0 < q < 1 \implies p^k > 0, q^{n-k} > 0 \implies p_X(k;n,p) > 0$

- Universal Summation of Unity:

$$\sum_{k \in \text{Supp}(X)} p_X(k;n,p) = \sum_{k=0}^{n} \binom{n}{k} p^k q^{n-k} \overset{(*)}{=} (p+q)^n = 1^n = 1$$

$(*)$ Binomial Theorem:

$$(x+y)^n = x^n + \binom{n}{1} x^{n-1} y + \binom{n}{2} x^{n-2} y^2 + \cdots + \binom{n}{n-1} x y^{n-1} + y^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

Mean of Binomial(n, p) random variable (Proof)

Let random variable $X \sim$ Binomial(n, p) s.t. $n \geq 1$, $0 < p < 1$ and $q := 1 - p$.
Then:

$$\mathbb{E}[X] = \sum_{k \in \text{Supp}(X)} k \cdot p_X(k; n, p) = \sum_{k=0}^{n} k \cdot \binom{n}{k} p^k q^{n-k} = p \sum_{k=0}^{n} k \cdot \binom{n}{k} p^{k-1} q^{n-k}$$

$$= p \sum_{k=0}^{n} \frac{\partial}{\partial p} \left[\binom{n}{k} p^k q^{n-k} \right] \overset{(*)}{=} p \frac{\partial}{\partial p} \left[\sum_{k=0}^{n} \binom{n}{k} p^k q^{n-k} \right] = p \frac{\partial}{\partial p} \left[(p+q)^n \right]$$

$$= p \cdot n(p+q)^{n-1} \frac{\partial}{\partial p} \left[p + q \right] = np(p+q)^{n-1}(1+0) = np \cdot 1^{n-1} \cdot 1 = np$$

$\therefore \quad \mathbb{E}[X] = np$ QED

$(*)$ Interchanging summation & differentiation works here, but not in general.
Take **Advanced Calculus** for the painful details.

Variance of Binomial(n,p) random variable (Proof)

Let random variable $X \sim$ Binomial(n,p) s.t. $n \geq 1$, $0 < p < 1$ and $q := 1 - p$.

$$\mathbb{E}[X^2] = \sum_{k \in \text{Supp}(X)} k^2 \cdot p_X(k;n,p) = \sum_{k=0}^{n} k^2 \cdot \binom{n}{k} p^k q^{n-k} = \sum_{k=1}^{n} k^2 \cdot \binom{n}{k} p^k q^{n-k}$$

$$\overset{(*)}{=} \sum_{k=1}^{n} nk \binom{n-1}{k-1} p^k q^{n-k} \overset{CV}{=} \sum_{j=0}^{n-1} n(j+1) \binom{n-1}{j} p^{j+1} q^{n-(j+1)}$$

$$= \sum_{j=0}^{n-1} nj \binom{n-1}{j} p^{j+1} q^{n-(j+1)} + \sum_{j=0}^{n-1} n \binom{n-1}{j} p^{j+1} q^{n-(j+1)}$$

$$(*) \quad k^2 \binom{n}{k} = k^2 \frac{n!}{k!(n-k)!} = nk \frac{(n-1)!}{(k-1)![(n-1)-(k-1)]!} = nk \binom{n-1}{k-1}$$

CV: Let $j = k - 1 \iff k = j + 1$

Then
$$\begin{aligned} k = n &\implies j+1 = n \implies j = n-1 \\ k = 1 &\implies j+1 = 1 \implies j = 1 - 1 = 0 \end{aligned}$$

Variance of Binomial(n,p) random variable (Proof)

Let random variable $X \sim$ Binomial(n,p) s.t. $n \geq 1,\ 0 < p < 1$ and $q := 1 - p$.

$$\begin{aligned}
\mathbb{E}[X^2] &= \sum_{j=0}^{n-1} nj\binom{n-1}{j}p^{j+1}q^{n-(j+1)} + \sum_{j=0}^{n-1} n\binom{n-1}{j}p^{j+1}q^{n-(j+1)} \\
&= np^2\sum_{j=0}^{n-1} j\binom{n-1}{j}p^{j-1}q^{(n-1)-j} + np\sum_{j=0}^{n-1}\binom{n-1}{j}p^jq^{(n-1)-j} \\
&= np^2\sum_{j=0}^{n-1}\frac{\partial}{\partial p}\left[\binom{n-1}{j}p^jq^{(n-1)-j}\right] + np\sum_{j=0}^{n-1}\binom{n-1}{j}p^jq^{(n-1)-j} \\
&= np^2\frac{\partial}{\partial p}\left[\sum_{j=0}^{n-1}\binom{n-1}{j}p^jq^{(n-1)-j}\right] + np\sum_{j=0}^{n-1}\binom{n-1}{j}p^jq^{(n-1)-j} \\
&\overset{(*)}{=} np^2\frac{\partial}{\partial p}\left[(p+q)^{n-1}\right] + np(p+q)^{n-1}
\end{aligned}$$

$(*)$ Binomial Theorem

Variance of Binomial(n, p) random variable (Proof)

Let random variable $X \sim$ Binomial(n, p) s.t. $n \geq 1,\ 0 < p < 1$ and $q := 1 - p$.

$$\begin{aligned}
\mathbb{E}[X^2] &= np^2 \frac{\partial}{\partial p}\left[(p+q)^{n-1}\right] + np(p+q)^{n-1} \\
&= np^2 \cdot (n-1)(p+q)^{n-2} \frac{\partial}{\partial p}\left[p+q\right] + np(p+q)^{n-1} \\
&= n(n-1)p^2 \cdot 1^{n-2} \cdot (1+0) + np \cdot 1^{n-1} \\
&= n(n-1)p^2 + np \\
&= n^2p^2 - np^2 + np
\end{aligned}$$

$\mathbb{V}[X] = \mathbb{E}[X^2] - (\mathbb{E}[X])^2 = (n^2p^2 - np^2 + np) - (np)^2 = np - np^2 = np(1-p) = npq$

QED

Probabilities of Binomial rv's via Binomial cdf Bi$(x; n, p)$

It can get quite tedious using the Binomial pmf.
It's easier to use the Binomial cdf, but the cdf has no elementary closed-form!
Instead, tables of numerical values of the Binomial cdf are used instead.

Probabilities of Binomial rv's via Binomial cdf Bi$(x; n, p)$

$\mathbf{n = 5}$	Success Probability (p)										
x	**0.1**	**0.2**	**0.25**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.75**	**0.8**	**0.9**
0	0.59049	0.32768	0.23730	0.16807	0.07776	0.03125	0.01024	0.00243	0.00098	0.00032	0.00001
1	0.91854	0.73728	0.63281	0.52822	0.33696	0.18750	0.08704	0.03078	0.01562	0.00672	0.00046
2	0.99144	0.94208	0.89648	0.83692	0.68256	0.50000	0.31744	0.16308	0.10352	0.05792	0.00856
3	0.99954	0.99328	0.98438	0.96922	0.91296	0.81250	0.66304	0.47178	0.36719	0.26272	0.08146
4	0.99999	0.99968	0.99902	0.99757	0.98976	0.96875	0.92224	0.83193	0.76270	0.67232	0.40951
5	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

Suppose $X \sim$ Binomial$(n = 5, p = 0.7)$. Then:

$$
\begin{aligned}
\mathbb{P}(X \leq 3) &= ?????? \\
\mathbb{P}(X \leq 4) &= ?????? \\
\mathbb{P}(X = 4) &= ?????? \\
\mathbb{P}(X > 4) &= ??????
\end{aligned}
$$

Probabilities of Binomial rv's via Binomial cdf Bi$(x; n, p)$

n = 5	Success Probability (p)										
x	**0.1**	**0.2**	**0.25**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.75**	**0.8**	**0.9**
0	0.59049	0.32768	0.23730	0.16807	0.07776	0.03125	0.01024	0.00243	0.00098	0.00032	0.00001
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3	0.99954	0.99328	0.98438	0.96922	0.91296	0.81250	0.66304	0.47178	0.36719	0.26272	0.08146
4	0.99999	0.99968	0.99902	0.99757	0.98976	0.96875	0.92224	0.83193	0.76270	0.67232	0.40951
5	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

Suppose $X \sim$ Binomial$(n = 5, p = 0.7)$. Then:

$$\mathbb{P}(X \leq 3) = \text{Bi}(3; 5, 0.7) \overset{LOOKUP}{=} \boxed{0.47178}$$
$$\mathbb{P}(X \leq 4) = \text{Bi}(4; 5, 0.7) \overset{LOOKUP}{=} \boxed{0.83193}$$
$$\mathbb{P}(X = 4) = \mathbb{P}(X \leq 4) - \mathbb{P}(X \leq 3) = 0.83193 - 0.47178 = \boxed{0.36015}$$
$$\mathbb{P}(X > 4) = 1 - \mathbb{P}(X \leq 4) = 1 - 0.83193 = \boxed{0.16807}$$

Probabilities of Binomial rv's via Binomial cdf Bi$(x; n, p)$

n = 5	**Success Probability** (p)										
x	**0.1**	**0.2**	**0.25**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.75**	**0.8**	**0.9**
0	0.59049	0.32768	0.23730	0.16807	0.07776	0.03125	0.01024	0.00243	0.00098	0.00032	0.00001
1	0.91854	0.73728	0.63281	0.52822	0.33696	0.18750	0.08704	0.03078	0.01562	0.00672	0.00046
2	0.99144	0.94208	0.89648	0.83692	0.68256	0.50000	0.31744	0.16308	0.10352	0.05792	0.00856
3	0.99954	0.99328	0.98438	0.96922	0.91296	0.81250	0.66304	0.47178	0.36719	0.26272	0.08146
4	0.99999	0.99968	0.99902	0.99757	0.98976	0.96875	0.92224	0.83193	0.76270	0.67232	0.40951
5	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

How, if possible, to compute Bi$(4; 5, 0.7)$ via calculator or software??

TI-82/83/84+	`binomcdf(5,0.7,4)`	`2nd` $\rightarrow$ `VARS`
TI-86	(Not Possible)	(Not Possible)
TI-89	(Not Possible)	(Not Possible)
TI-36X Pro	Binomialcdf	`2nd` $\rightarrow$ `data`
MATLAB	`binocdf(4,5,0.7)`	(Stats Toolbox)
R	`pbinom(q=4,size=5,prob=0.7)`	
Python	`scipy.stats.binom.cdf(4,5,0.7)`	(Needs SciPy)

Probabilities of Binomial rv's via Binomial cdf Bi$(x; n, p)$

n = 10	**Success Probability** (p)										
x	**0.1**	**0.2**	**0.25**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.75**	**0.8**	**0.9**
0	0.34868	0.10737	0.05631	0.02825	0.00605	0.00098	0.00010	0.00001	0.00000	0.00000	0.00000
1	0.73610	0.37581	0.24403	0.14931	0.04636	0.01074	0.00168	0.00014	0.00003	0.00000	0.00000
2	0.92981	0.67780	0.52559	0.38278	0.16729	0.05469	0.01229	0.00159	0.00042	0.00008	0.00000
3	0.98720	0.87913	0.77588	0.64961	0.38228	0.17187	0.05476	0.01059	0.00351	0.00086	0.00001
4	0.99837	0.96721	0.92187	0.84973	0.63310	0.37695	0.16624	0.04735	0.01973	0.00637	0.00015
5	0.99985	0.99363	0.98027	0.95265	0.83376	0.62305	0.36690	0.15027	0.07813	0.03279	0.00163
6	0.99999	0.99914	0.99649	0.98941	0.94524	0.82812	0.61772	0.35039	0.22412	0.12087	0.01280
7	1.00000	0.99992	0.99958	0.99841	0.98771	0.94531	0.83271	0.61722	0.47441	0.32220	0.07019
8	1.00000	1.00000	0.99997	0.99986	0.99832	0.98926	0.95364	0.85069	0.75597	0.62419	0.26390
9	1.00000	1.00000	1.00000	0.99999	0.99990	0.99902	0.99395	0.97175	0.94369	0.89263	0.65132
10	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

Probabilities of Binomial rv's via Binomial cdf Bi$(x; n, p)$

n = 15	**Success Probability** (p)										
x	**0.1**	**0.2**	**0.25**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.75**	**0.8**	**0.9**
0	0.20589	0.03518	0.01336	0.00475	0.00047	0.00003	0.00000	0.00000	0.00000	0.00000	0.00000
1	0.54904	0.16713	0.08018	0.03527	0.00517	0.00049	0.00003	0.00000	0.00000	0.00000	0.00000
2	0.81594	0.39802	0.23609	0.12683	0.02711	0.00369	0.00028	0.00001	0.00000	0.00000	0.00000
3	0.94444	0.64816	0.46129	0.29687	0.09050	0.01758	0.00193	0.00009	0.00001	0.00000	0.00000
4	0.98728	0.83577	0.68649	0.51549	0.21728	0.05923	0.00935	0.00067	0.00012	0.00001	0.00000
5	0.99775	0.93895	0.85163	0.72162	0.40322	0.15088	0.03383	0.00365	0.00079	0.00011	0.00000
6	0.99969	0.98194	0.94338	0.86886	0.60981	0.30362	0.09505	0.01524	0.00419	0.00078	0.00000
7	0.99997	0.99576	0.98270	0.94999	0.78690	0.50000	0.21310	0.05001	0.01730	0.00424	0.00003
8	1.00000	0.99922	0.99581	0.98476	0.90495	0.69638	0.39019	0.13114	0.05662	0.01806	0.00031
9	1.00000	0.99989	0.99921	0.99635	0.96617	0.84912	0.59678	0.27838	0.14837	0.06105	0.00225
10	1.00000	0.99999	0.99988	0.99933	0.99065	0.94077	0.78272	0.48451	0.31351	0.16423	0.01272
11	1.00000	1.00000	0.99999	0.99991	0.99807	0.98242	0.90950	0.70313	0.53871	0.35184	0.05556
12	1.00000	1.00000	1.00000	0.99999	0.99972	0.99631	0.97289	0.87317	0.76391	0.60198	0.18406
13	1.00000	1.00000	1.00000	1.00000	0.99997	0.99951	0.99483	0.96473	0.91982	0.83287	0.45096
14	1.00000	1.00000	1.00000	1.00000	1.00000	0.99997	0.99953	0.99525	0.98664	0.96482	0.79411
15	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

Textbook Logistics for Section 3.4

- Difference(s) in Notation:

CONCEPT	TEXTBOOK NOTATION	SLIDES/OUTLINE NOTATION
Probability of Event	$P(A)$	$\mathbb{P}(A)$
Measure of Event	$N(A)$	$\lvert A \rvert$
Support of a r.v.	"All possible values of X"	Supp(X)
Support of a r.v.	D	Supp(X)
pmf of a r.v.	$p(x)$	$p_X(k)$
cdf of a r.v.	$F(x)$	$F_X(x)$
Expected Value of a r.v.	$E(X)$	$\mathbb{E}[X]$
Variance of a r.v.	$V(X)$	$\mathbb{V}[X]$
Binomial pmf	$b(x;n,p)$	$p_X(k;n,p)$
Binomial cdf	$B(x;n,p)$	Bi$(x;n,p)$

- Ignore Rule of Thumb for Deciding whether a "without-replacement" experiment can be treated as being Binomial (pg 119)

Fin.