Uniform & Normal Distributions

Engineering Statistics
Section 4.3

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TTU

04 March 2016

PART I:

UNIFORM DISTRIBUTION

Uniform Random Variables (Applications)

Uniform random variables reasonably model:

- Waiting times for a single arrival that repeats over a small time period:
  - Waiting time for a bus arrival over a 10-min interval
  - Waiting time for a train arrival over a 1-hour interval
- Randomly-chosen point on a line segment
- Random generation of numbers
  - This is how a computer randomly selects a number in an interval
- Very Small Measurement Errors:
  - Time-measuring test equipment is accurate to within 0.05 μsec
  - Quantization error in analog-to-digital signal conversion (ADC)

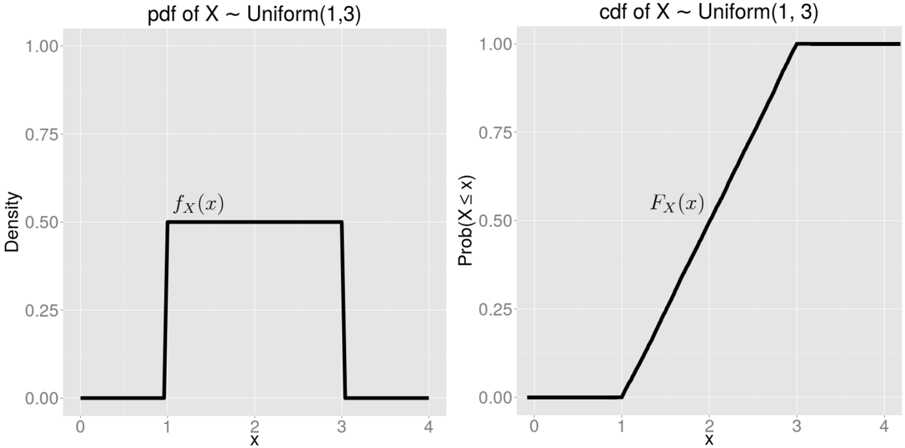
Uniform distributions are also used to develop later statistical methods.

Uniform Random Variables (Summary)

Proposition

Notation	$X \sim \textit{Uniform}(a, b), \quad a < b$
Parameter(s)	$a, b \in \mathbb{R}$
Support	$\textit{Supp}(X) = [a, b]$
pdf	$f_X(x; a, b) = \frac{1}{b-a}$
Mean	$\mathbb{E}[X] = \frac{1}{2}(b + a)$
Variance	$\mathbb{V}[X] = \frac{1}{12}(b - a)^2$
Model(s)	*Waiting time for a bus* *Random point on line segment* *Random-number generation* *Very small measurement errors*

Uniform Density Plots (pdf & cdf)



Verification that Uniform(a, b) pdf truly is a valid pdf

$$X \sim \text{Uniform}(a,b) \iff f_X(x;a,b) = \begin{cases} \dfrac{1}{b-a} & \text{, for } a \le x \le b \\ 0 & \text{elsewhere} \end{cases}$$

- Non-negativity on its support:

  Observe that $a < b \implies b - a > 0 \implies \frac{1}{b-a} > 0$
  Hence, $f_X(x;a,b) \ge 0$

- Universal Integral of Unity:

$$\int_{-\infty}^{\infty} f_X(x)\,dx = \int_a^b \frac{1}{b-a}\,dx = \left[\frac{x}{b-a}\right]_{x=a}^{x=b} \overset{FTC}{=} \frac{b}{b-a} - \frac{a}{b-a} = \frac{b-a}{b-a} = 1$$

Mean of Uniform(a, b) random variable (Proof)

Let random variable $X \sim$ Uniform(a, b), where $a < b$. Then:

$$\mathbb{E}[X] = \int_{-\infty}^{\infty} x \cdot f_X(x; a, b) = \int_a^b \frac{x}{b-a}\, dx = \left[\frac{x^2}{2(b-a)}\right]_{x=a}^{x=b}$$

$$\overset{FTC}{=} \frac{(b)^2}{2(b-a)} - \frac{(a)^2}{2(b-a)} = \frac{b^2 - a^2}{2(b-a)} = \frac{(b+a)(b-a)}{2(b-a)} = \frac{b+a}{2}$$

$\therefore \quad \mathbb{E}[X] = \frac{1}{2}(a+b)$ QED

Variance of Uniform(a, b) random variable (Proof)

Let random variable $X \sim$ Uniform(a, b), where $a < b$. Then:

$$\mathbb{E}[X^2] = \int_{-\infty}^{\infty} x^2 \cdot f_X(k; a, b) = \int_a^b \frac{x^2}{b-a}\, dx = \left[\frac{x^3}{3(b-a)}\right]_{x=a}^{x=b}$$

$$\overset{FTC}{=} \frac{(b)^3 - (a)^3}{3(b-a)} = \frac{(b-a)(b^2+ba+a^2)}{3(b-a)} = \frac{a^2+ab+b^2}{3}$$

$$\begin{aligned} \mathbb{V}[X] &= \mathbb{E}[X^2] - (\mathbb{E}[X])^2 = \frac{a^2+ab+b^2}{3} - \left(\frac{a+b}{2}\right)^2 \\ &= \frac{a^2+ab+b^2}{3} - \frac{a^2+2ab+b^2}{4} = \frac{4(a^2+ab+b^2)}{12} - \frac{3(a^2+2ab+b^2)}{12} \\ &= \frac{4a^2+4ab+4b^2-3a^2-6ab-3b^2}{12} = \frac{a^2-2ab+b^2}{12} = \frac{(b-a)^2}{12} \end{aligned}$$

$\therefore \quad \mathbb{V}[X] = \frac{1}{12}(b-a)^2$ QED

PART II:

NORMAL DISTRIBUTION

Carl Friedrich Gauss (1777-1855)

Gauss made fundamental developments with the normal distribution. Normal distributions are sometimes referred to **Gaussian** distributions.

Normal Random Variables (Applications)

Normal random variables reasonably model:

- First midterm scores of a lecture hall class of students
- Grade point averages of college students
- Heights of people at a large busy conference
- Monthly rainfall totals in a humid climate
- Measurement errors
- Brownian motion of particles suspended in fluid

Normal distributions are often used to develop later statistical methods, many of which will be encountered throughout the course. (Ch5 - Ch9)

Normal Random Variables (Summary)

Proposition

Notation	$X \sim \textit{Normal}(\mu, \sigma^2), \ \ \mu \in \mathbb{R}, \ \sigma^2 > 0$
Parameter(s)	$\mu \equiv$ *Mean* $\sigma^2 \equiv$ *Variance*
Support	$\textit{Supp}(X) = (-\infty, \infty)$
pdf	$f_X(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/(2\sigma^2)}$
Mean	$\mathbb{E}[X] = \mu$
Variance	$\mathbb{V}[X] = \sigma^2$
Model(s)	*Exam Scores, Heights, Measurement Errors*
Assumption(s)	*1. Very large population size.*

Standard Normal Random Variables (Summary)

A **standard normal** r.v., usually denoted Z, is used to actually compute probabilities of Normal(μ, σ^2) r.v.'s via its cdf which is denoted $\Phi(z)$.

Proposition

Notation	$Z \sim$ *Normal*$(0,1)$
Parameter(s)	(*None*)
Support	*Supp*$(Z) = (-\infty, \infty)$
pdf	$f_Z(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$
cdf	$\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\xi^2/2}\, d\xi$
Mean	$\mathbb{E}[Z] = 0$
Variance	$\mathbb{V}[Z] = 1$
Model(s)	
Assumption(s)	*1. Very large population size*

Mean of Normal$(0, 1)$ random variable (Proof)

Let $Z \sim$ Normal$(0,1)$ $\iff$ pdf $f_Z(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$

Then:

$$\begin{aligned}
\mathbb{E}[Z] &= \int_{-\infty}^{\infty} z \cdot f_Z(z)\, dz = \int_{-\infty}^{\infty} z \cdot \frac{1}{\sqrt{2\pi}} e^{-z^2/2}\, dz = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z e^{-z^2/2}\, dz \\
&= \frac{1}{\sqrt{2\pi}} \left[e^{-z^2/2} \right]_{z\to-\infty}^{z\to\infty} \overset{FTC}{=} \frac{1}{\sqrt{2\pi}} \left[\left(\lim_{z\to\infty} e^{-z^2/2} \right) - \left(\lim_{z\to-\infty} e^{-z^2/2} \right) \right] \\
&= \frac{1}{\sqrt{2\pi}} \left[0 - 0 \right] = 0
\end{aligned}$$

$\therefore \quad \mathbb{E}[Z] = 0$ QED

Variance of Normal$(0, 1)$ random variable (Proof)

Let $Z \sim$ Normal$(0,1)$ $\iff$ pdf $f_Z(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$

$$\mathbb{E}[Z^2] = \int_{-\infty}^{\infty} z^2 \cdot f_Z(z)\, dz = \int_{-\infty}^{\infty} z^2 \cdot \frac{1}{\sqrt{2\pi}} e^{-z^2/2}\, dz = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z^2 e^{-z^2/2}\, dz$$

$$\overset{IBP}{=} \frac{1}{\sqrt{2\pi}} \left[\left[-ze^{-z^2/2} \right]_{z\to-\infty}^{z\to\infty} - \int_{-\infty}^{\infty} \left(-e^{-z^2/2} \right) dz \right]$$

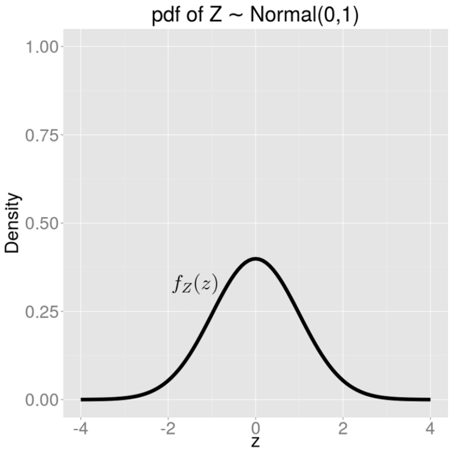
$$\overset{FTC}{=} \frac{1}{\sqrt{2\pi}} \left[\left[0 - 0 \right] + \int_{-\infty}^{\infty} e^{-z^2/2}\, dz \right] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-z^2/2}\, dz$$

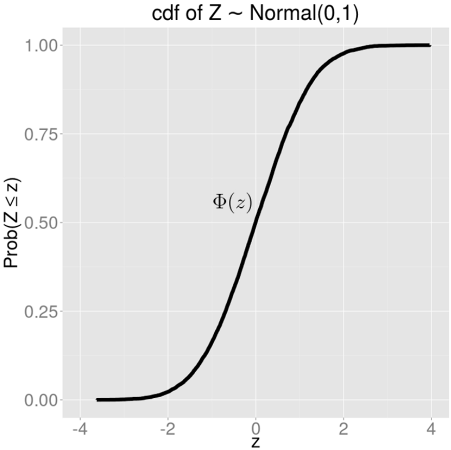
$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-z^2/2}\, dz = \int_{-\infty}^{\infty} f_Z(z)\, dz = 1$$

$\therefore\ \mathbb{V}[Z] = \mathbb{E}[Z^2] - (\mathbb{E}[Z])^2 = 1 - (0)^2 = 1$ QED

(IBP): Let $\left\{ \begin{array}{rcl} u & = & z \\ dv & = & ze^{-z^2/2}\, dz \end{array} \right. \implies \left\{ \begin{array}{rcl} du & = & dz \\ v & = & -e^{-z^2/2} \end{array} \right.$

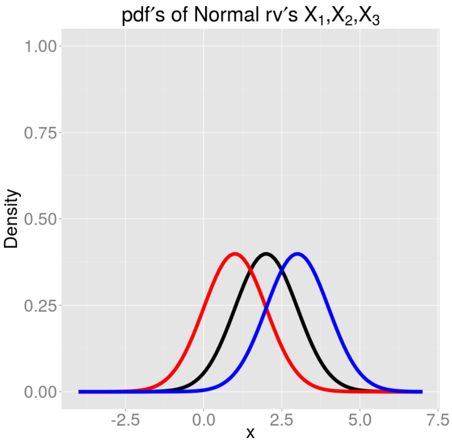
Standard Normal Density Plots (pdf & cdf)





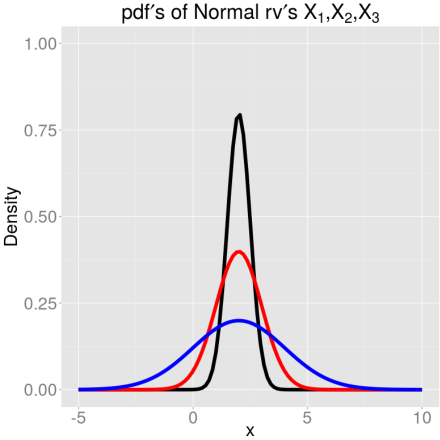
Normal pdf's have this distinctive **bell-shaped** curve.

How changing μ affects the **location** of a Normal pdf



Here:
$X_1 \sim$ Normal$(1, 1)$ (Red Curve)
$X_2 \sim$ Normal$(2, 1)$ (Black Curve)
$X_3 \sim$ Normal$(3, 1)$ (Blue Curve)

How changing σ^2 affects the **shape** of a Normal pdf



Here:
$X_1 \sim$ Normal$(1, 0.25)$ (Black Curve)
$X_2 \sim$ Normal$(1, 1)$ (Red Curve)
$X_3 \sim$ Normal$(1, 4)$ (Blue Curve)

Properties of the Normal Distribution

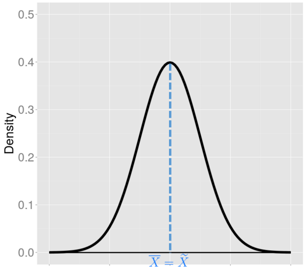
The normal distribution has some very properties that many other discrete and continuous distributions do not possess:

Proposition

(Properties of the Normal Distribution)

Let random variable $X \sim$ Normal(μ, σ^2). Then the following are all true:

- *The density curve is* ***unimodal, bell-shaped, and not skewed****.*
- *The density curve is* ***symmetric*** *about its* ***mean*** $\overline{X}$.
- *The mean & median of X are equal:* $\overline{X} = \widetilde{X}$



Computing Probabilities involving Normal r.v.'s

Let $X \sim \text{Normal}(\mu, \sigma^2) \iff$ pdf $f_X(x) = \dfrac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/(2\sigma^2)}$

Then, $\mathbb{P}(a \le X \le b) = \displaystyle\int_a^b f_X(x)\, dx = \frac{1}{\sigma\sqrt{2\pi}} \underbrace{\int_a^b e^{-(x-\mu)^2/(2\sigma^2)}\, dx}_{\text{Nonelementary integral!!}}$

and the cdf is $F_X(x) = \mathbb{P}(X \le x) = \dfrac{1}{\sigma\sqrt{2\pi}} \underbrace{\int_{-\infty}^x e^{-(\xi-\mu)^2/(2\sigma^2)}\, d\xi}_{\text{Nonelementary integral!!}}$

The problem with the Normal distribution is the resulting integrals have no finite closed-form anti-derivative!!

The fix to this is to **numerically** approximate the Normal **cdf** via a **table**. But since μ & σ are real numbers, there would be infinitely many tables!!

The solution is to **convert** Normal(μ, σ^2) r.v. X to **standard normal** r.v. Z, and then use the table for the **standard normal cdf**, which is denoted $\Phi(z)$.

Converting Normal r.v. $\to$ Standard Normal r.v.

Converting a Normal r.v. to a standard Normal r.v. is called **standardization**:

Proposition

(Standardizing Normal(μ, σ) *r.v.* $\to$ *Normal*$(0, 1)$ *r.v.)*

Let random variable $X \sim$ *Normal*(μ, σ^2)*.*
Then, X *can be* ***standardized*** *to the standard normal r.v.* $Z \sim$ *Normal*$(0, 1)$ *by*

$$Z = \frac{X - \mu}{\sigma}$$

PROOF: Let $X \sim$ Normal$(\mu, \sigma^2) \implies F_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} e^{-(\xi-\mu)^2/(2\sigma^2)} \, d\xi$

(CV): Let $\widetilde{\xi} = \frac{\xi - \mu}{\sigma} \implies d\widetilde{\xi} = \frac{1}{\sigma} d\xi \implies d\xi = \sigma \, d\widetilde{\xi} \implies \begin{cases} \widetilde{\xi}(\mu + \sigma z) = z \\ \widetilde{\xi}(-\infty) = -\infty \end{cases}$

$$\therefore \; F_Z(z) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\mu+\sigma z} e^{-(\xi-\mu)^2/(2\sigma^2)} \, d\xi \overset{CV}{=} \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\widetilde{\xi}^2/2} (\sigma \, d\widetilde{\xi})$$

$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\widetilde{\xi}^2/2} \, d\widetilde{\xi} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\xi^2/2} \, d\xi \longleftarrow$ cdf of Normal(0,1) QED

Converting Standard Normal r.v. $\rightarrow$ Normal r.v.

The reverse conversion is just as straightforward to perform:

Proposition

(Converting Normal$(0, 1)$ r.v. $\rightarrow$ Normal(μ, σ^2) r.v.)

Let random variable $Z \sim$ Normal$(0, 1)$.
Then, Z can be converted to the normal r.v. $X \sim$ Normal(μ, σ^2) by

$$X = \mu + \sigma Z$$

Probabilities of Normal r.v.'s via Std Normal cdf $\Phi(z)$

Proposition

(Computing Probabilities of Normal(μ, σ^2) r.v. via Standard Normal cdf $\Phi(z)$)

Let random variable $X \sim$ Normal(μ, σ^2). Then:

$$\mathbb{P}(X \leq a) = \mathbb{P}\left(Z \leq \frac{a-\mu}{\sigma}\right) = \Phi\left(\frac{a-\mu}{\sigma}\right)$$

$$\mathbb{P}(a \leq X \leq b) = \mathbb{P}\left(\frac{a-\mu}{\sigma} \leq Z \leq \frac{b-\mu}{\sigma}\right) = \Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right)$$

$$\mathbb{P}(X \geq b) = \mathbb{P}\left(Z \geq \frac{b-\mu}{\sigma}\right) = 1 - \Phi\left(\frac{b-\mu}{\sigma}\right)$$

Computing Standard Normal cdf $\Phi(z)$ via Table

The **standard normal table** below approximates $\Phi(z) = \mathbb{P}(Z \leq z)$:

z	+0.00	+0.01	+0.02	+0.03	+0.04	+0.05	+0.06	+0.07	+0.08	+0.09
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53980	0.54380	0.54776	0.55172	0.55567	0.55966	0.56360	0.56749	0.57142	0.57535
0.2	0.57930	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91308	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900

Computing $\Phi(z)$ via Table or Calculator or Software

z	+0.00	+0.01	+0.02	+0.03	+0.04	+0.05	+0.06	+0.07	+0.08	+0.09
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53980	0.54380	0.54776	0.55172	0.55567	0.55966	0.56360	0.56749	0.57142	0.57535
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0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
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1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
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1.4	0.91924	0.92073	0.92220	0.92364	[illegible]	0.92647	0.92785	0.92922	0.93056	0.93189
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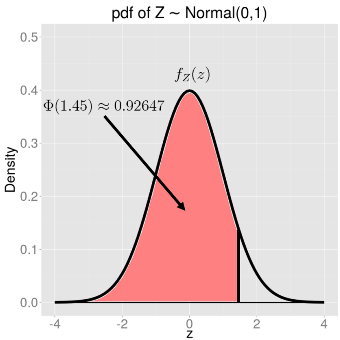
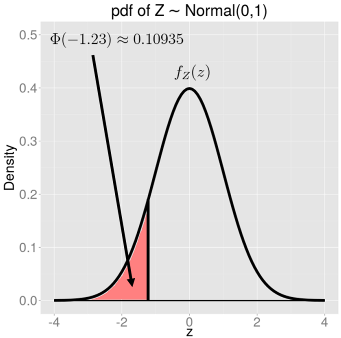


TI-82/83/84+	`normalcdf(−1E99,1.45,0,1)`	`2nd` → `VARS`
TI-86	`nmcdf(−1E99,1.45,0,1)`	`2nd` → `MATH`
TI-89	Normal cdf	`APPS` → Stats
TI-36X Pro	Normalcdf	`2nd` → `data`
MATLAB	`normcdf(1.45)`	(Stats Toolbox)
R	`pnorm(1.45)`	
Python	`scipy.stats.norm.cdf(1.45)`	(Needs SciPy)

Computing $\Phi(z)$ via Table or Calculator or Software

$$\Phi(-1.23) = 1 - \Phi(1.23) \approx 1 - 0.89065 = 0.10935$$

z	+0.00	+0.01	+0.02	+0.03	+0.04	+0.05	+0.06	+0.07	+0.08	+0.09
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53980	0.54380	0.54776	0.55172	0.55567	0.55966	0.56360	0.56749	0.57142	0.57535
0.2	0.57930	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91308	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408



TI-82/83/84+
TI-86
TI-89
	`normalcdf(-1E99,-1.23,0,1)`
	`nmcdf(-1E99,-1.23,0,1)`
	Normal cdf
		2nd → VARS
		2nd → MATH
		APPS → Stats

TI-36X Pro	Normalcdf	2nd → data

MATLAB
R
Python
	`normcdf(-1.23)`
	`pnorm(-1.23)`
	`scipy.stats.norm.cdf(-1.23)`
		(Stats Toolbox)
		
		(Needs SciPy)

- Difference(s) in Notation:

CONCEPT	TEXTBOOK NOTATION	SLIDES/OUTLINE NOTATION
Probability of Event	$P(A)$	$\mathbb{P}(A)$
Support of a r.v.	"All possible values of X"	Supp(X)
pdf of a r.v.	$f(x)$	$f_X(x)$
cdf of a r.v.	$F(x)$	$F_X(x)$
Expected Value of r.v.	$E(X)$	$\mathbb{E}[X]$
Variance of r.v.	$V(X)$	$\mathbb{V}[X]$
Normal Distribution	$N(\mu, \sigma^2)$	Normal(μ, σ^2)
Normal Distribution	$N(3, 4)$	Normal$(\mu = 3, \sigma^2 = 4)$
Normal Distribution	$N(\mu = 3, \sigma = 4)$	Normal$(\mu = 3, \sigma = 4)$
Standard Normal	$N(0, 1)$	Normal$(0, 1)$

Textbook Logistics for Section 4.3

- Skip the "z_α Notation for z-Critical Values" section (pg 160-161)
  - This is important for Statistical Inference.
  - Hence, z_α will be covered starting in Chapter 7.
- Skip the **Empirical Rule** (middle blue box on pg 163)
  - This is a nice handy rule, but it's not useful now.
  - The Empirical Rule will be covered in Chapter 5.
- Skip the "Approximating the Binomial Distribution" section (pg 165-166)
  - This will be covered in Chapter 5.

Fin.