EX 10.2.1: Dentists use resin composites and ceramic fillings among others for cavities in teeth. The shear bond strengths of resin composite-ceramic bonds formed from three possible configurations (conventional, all-composite, reversed) were measured (in MPa) and summarized in the following table:

GROUP:	SAMPLE SIZE:	MEAN:	STD DEV:
Conventional	11	$\overline{x}_{1\bullet} = 10.37$	$s_1 = 1.99$
All-Composite	11	$\overline{x}_{2\bullet} = 20.12$	$s_2 = 2.45$
Reversed	11	$\overline{x}_{3\bullet} = 18.02$	$s_3 = 2.52$

A similar table and all the details regarding the experiment can be found in the following paper:

> A. Della Bona, R. van Noort, "Shear vs. Tensile Bond Strength of Resin Composite Bonded to Ceramic", *Journal of Dental Research*, **74** (1995), 1591-1596.

A 1-Factor ANOVA at significance level $\alpha = 0.05$ was performed, resulting in the rejection of the null hypothesis. The error mean square was found to be $\text{MS}_{res} \approx 5.4377$ during the ANOVA procedure.

(a) Perform the Tukey Complete Pairwise Post-Hoc Comparison to determine which groups significantly differ.

1^{st}, compute the significant difference width:

$$\nu_{res} := I(J-1) = 3 \cdot (11-1) = 30$$
$$w = q^*_{I,\nu_{res};\alpha} \cdot \sqrt{\text{MS}_{res}/J} \approx q^*_{3,30;0.05} \cdot \sqrt{5.4377/11} \overset{LOOKUP}{\approx} \mathbf{3.49} \cdot \sqrt{5.4377/11} \approx 2.45$$

2^{nd}, sort the treatment means in ascending order: $\overline{x}_{(1)\bullet} \le \overline{x}_{(2)\bullet} \le \overline{x}_{(3)\bullet}$

$\overline{x}_{(1)\bullet}$	$\overline{x}_{(2)\bullet}$	$\overline{x}_{(3)\bullet}$
$\overline{x}_{1\bullet}$	$\overline{x}_{3\bullet}$	$\overline{x}_{2\bullet}$
10.37	18.02	20.12

3^{rd}, underline neighboring treatment means that are within a distance of $w \approx 2.45$ from each other:

$\overline{x}_{1\bullet}$	$\overline{x}_{3\bullet}$	$\overline{x}_{2\bullet}$
10.37	18.02	20.12

4^{th}, interpret the underline(s):

> The experiment suggests that the all-composite and reversed configurations each have a significantly higher shear bond strength than the conventional.
>
> The experiment suggests that there is not a significant difference in shear bond strength between the all-composite and reversed configurations.

(b) Compute the 95% t-CI comparing the conventional config to the reversed and all-composite configurations.

μ_1 vs. $(\mu_2, \mu_3) \implies \sum_k c_k\mu_k = \mu_1 - \frac{1}{2}(\mu_2 + \mu_3) \implies \sum_k c_k\overline{x}_{k\bullet} = \overline{x}_{1\bullet} - \frac{1}{2}(\overline{x}_{2\bullet} + \overline{x}_{3\bullet}) = -8.7$

$t^*_{\nu_{res};\alpha/2} = t^*_{30;0.025} \overset{LOOKUP}{\approx} 2.042, \qquad \sum_k c_k^2 = c_1^2 + c_2^2 + c_3^2 = 1^2 + \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^2 = 1.5$

$\therefore$ 95% t-CI : $\sum_k c_k\overline{x}_{k\bullet} \pm t^*_{\nu_{res};\alpha/2} \cdot \sqrt{\text{MS}_{res} \cdot \sum_k c_k^2/J} \implies -8.7 \pm 1.5 \cdot \sqrt{5.4377 \cdot 1.5/11}$

$\implies -8.7 \pm 1.2917 \implies \boxed{(\mathbf{-9.9917},\ \mathbf{-7.4083})}$

The fact that the CI does not contain zero and is entirely negative indicates that the shear bond strength of the conventional configuration is significantly smaller than that of the (reversed,all-composite) group.