GOSSET'S Q DISTRIBUTION [DEVORE 10.2]

- **GOSSET'S Q DISTRIBUTION (DEFINITION):**

Notation	$Q \sim Q_{M,\nu}$
Parameters	$M \equiv$ # Groups/Cells/Treatments $\nu \equiv$ # degrees of freedom
Support	$\text{Supp}(Q) = [0, \infty)$
pdf	$f_Q(q; M, \nu) := \frac{\sqrt{2\pi} \cdot M(M-1)\nu^{\nu/2}}{\Gamma(\nu/2) \cdot 2^{-1+\nu/2}} \cdot \int_0^\infty x^\nu \cdot \Phi'(\sqrt{\nu} \cdot x) \cdot \left[\int_{-\infty}^{\infty} \Phi'(u)\Phi'(u - qx)[\Phi(u) - \Phi(u - qx)]^{M-2} du\right] dx$
Model(s)	(Used exclusively for Statistical Inference)

$\Phi(\cdot) \equiv$ Std Normal cdf $\quad \Phi'(\cdot) \equiv$ Std Normal pdf $\quad \Gamma(\cdot) \equiv$ Gamma Function

- **GOSSET'S Q DISTRIBUTION (DERIVATION):**

Given an experiment with M Normal(μ, σ^2) random samples each of size J.
Define $\overline{X}_{(1)}, \overline{X}_{(M)}$ to be the smallest and largest sample means, respectively.
Moreover, let S^2_{pool} be the pooled sample variance from the M samples.

Then, a Q **distribution** (AKA **Studentized Range distribution**) can created as follows:

$$Q := \frac{\overline{X}_{(M)} - \overline{X}_{(1)}}{S_{pool}/\sqrt{J}} \sim Q_{M,M(J-1)}$$

- **Q CUTOFFS (AKA Q CRITICAL VALUES):**

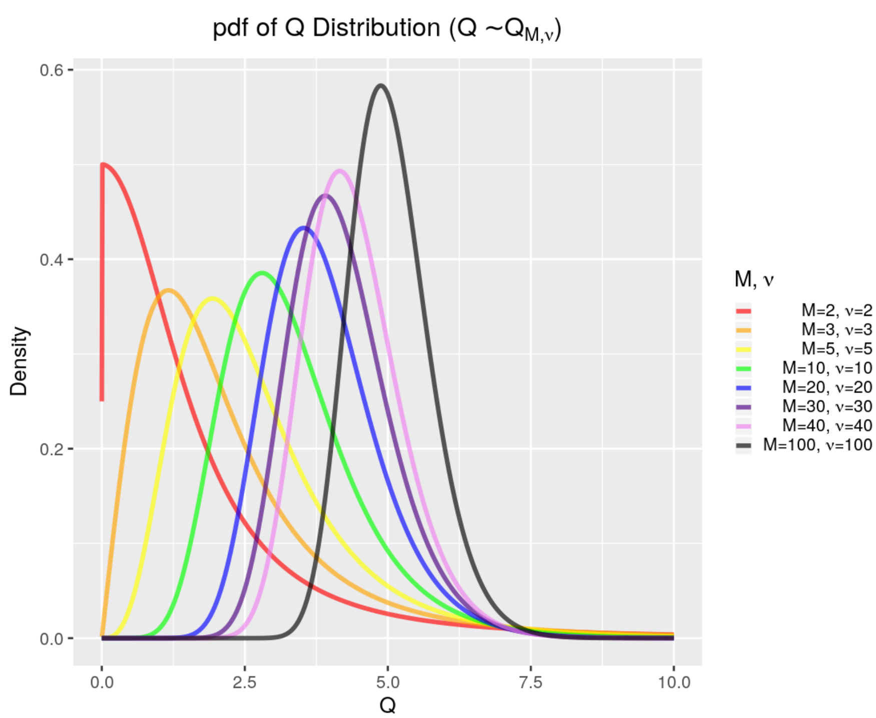
$q^*_{M,\nu;\alpha}$ is called a Q**-cutoff** of $Q_{M,\nu}$ distribution such that its upper-tail probability is its subscript value α:

$$\mathbb{P}(Q > q^*_{M,\nu;\alpha}) = \alpha$$

NOTE: Do not confuse Q-cutoff $q^*_{M,\nu;\alpha}$ with Q percentile $q_{M,\nu;\alpha}$: $\quad \mathbb{P}(Q \leq q_{M,\nu;\alpha}) = \alpha$

Another name for Q-cutoff is Q **critical value**.

- **GOSSET'S Q DISTRIBUTION (EXAMPLE PDF PLOTS):**



($\alpha = \mathbf{0.1}$) GOSSET'S Q-CUTOFFS, $q^*_{M,\nu;\alpha}$ $\mathbb{P}(Q > q^*_{M,\nu;\alpha}) = \alpha$

ν \ M	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
3	3.328	4.463	5.200	5.740	6.164	6.512	6.807	7.062	7.286	7.486	7.667	7.831	7.981	8.119
4	3.015	3.971	4.588	5.037	5.390	5.680	5.926	6.139	6.327	6.494	6.645	6.782	6.908	7.024
5	2.850	3.713	4.265	4.666	4.981	5.239	5.459	5.649	5.816	5.965	6.100	6.223	6.335	6.439
6	2.748	3.554	4.066	4.436	4.728	4.967	5.169	5.344	5.499	5.637	5.761	5.875	5.979	6.074
7	2.679	3.448	3.931	4.281	4.556	4.781	4.972	5.137	5.283	5.413	5.530	5.637	5.735	5.825
8	2.630	3.371	3.835	4.170	4.432	4.647	4.829	4.987	5.126	5.250	5.362	5.464	5.557	5.644
9	2.592	3.313	3.762	4.085	4.338	4.546	4.721	4.873	5.007	5.126	5.234	5.332	5.422	5.505
10	2.563	3.268	3.704	4.019	4.264	4.466	4.636	4.784	4.913	5.029	5.134	5.229	5.316	5.397
11	2.540	3.232	3.659	3.966	4.205	4.402	4.568	4.711	4.838	4.951	5.053	5.145	5.230	5.309
12	2.521	3.202	3.621	3.922	4.157	4.349	4.511	4.652	4.776	4.886	4.986	5.076	5.159	5.236
13	2.504	3.178	3.590	3.885	4.116	4.305	4.464	4.602	4.724	4.832	4.930	5.018	5.100	5.175
14	2.491	3.157	3.563	3.855	4.082	4.267	4.424	4.560	4.679	4.786	4.882	4.969	5.049	5.123
15	2.479	3.139	3.540	3.828	4.052	4.235	4.390	4.524	4.641	4.746	4.841	4.927	5.006	5.079
16	2.469	3.123	3.520	3.805	4.026	4.207	4.360	4.492	4.608	4.712	4.805	4.890	4.968	5.040
17	2.460	3.109	3.503	3.785	4.004	4.183	4.334	4.464	4.579	4.681	4.773	4.857	4.934	5.005
18	2.452	3.097	3.487	3.767	3.984	4.161	4.310	4.440	4.553	4.654	4.746	4.829	4.905	4.975
19	2.445	3.087	3.474	3.751	3.966	4.142	4.290	4.418	4.530	4.630	4.721	4.803	4.878	4.948
20	2.439	3.077	3.462	3.737	3.950	4.124	4.271	4.398	4.510	4.609	4.699	4.780	4.855	4.924
30	2.400	3.017	3.386	3.648	3.851	4.016	4.155	4.275	4.380	4.474	4.559	4.635	4.705	4.770

($\alpha = \mathbf{0.05}$) GOSSET'S Q-CUTOFFS, $q^*_{M,\nu;\alpha}$ $\mathbb{P}(Q > q^*_{M,\nu;\alpha}) = \alpha$

ν \ M	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
3	4.501	5.904	6.827	7.505	8.039	8.480	8.853	9.177	9.462	9.716	9.945	10.15	10.34	10.52
4	3.926	5.033	5.758	6.290	6.709	7.055	7.348	7.602	7.826	8.027	8.208	8.372	8.523	8.663
5	3.635	4.596	5.219	5.675	6.035	6.331	6.583	6.802	6.995	7.167	7.323	7.465	7.595	7.715
6	3.460	4.334	4.896	5.307	5.630	5.897	6.123	6.320	6.493	6.648	6.789	6.916	7.034	7.142
7	3.344	4.161	4.682	5.061	5.361	5.607	5.816	5.998	6.158	6.302	6.431	6.549	6.658	6.758
8	3.261	4.037	4.529	4.887	5.168	5.400	5.597	5.768	5.918	6.053	6.175	6.286	6.388	6.482
9	3.199	3.945	4.415	4.756	5.025	5.245	5.433	5.595	5.739	5.867	5.983	6.088	6.186	6.275
10	3.151	3.874	4.327	4.655	4.913	5.125	5.305	5.461	5.598	5.722	5.833	5.934	6.027	6.114
11	3.113	3.817	4.257	4.574	4.824	5.029	5.203	5.353	5.486	5.605	5.713	5.811	5.901	5.984
12	3.081	3.771	4.199	4.509	4.751	4.950	5.119	5.266	5.395	5.510	5.615	5.710	5.797	5.878
13	3.055	3.732	4.151	4.454	4.690	4.885	5.050	5.192	5.318	5.431	5.533	5.625	5.710	5.789
14	3.033	3.700	4.111	4.407	4.639	4.829	4.991	5.130	5.253	5.364	5.463	5.554	5.637	5.714
15	3.014	3.672	4.076	4.368	4.595	4.782	4.940	5.077	5.198	5.306	5.403	5.492	5.574	5.649
16	2.998	3.648	4.046	4.333	4.557	4.741	4.897	5.031	5.150	5.256	5.352	5.439	5.519	5.593
17	2.984	3.627	4.020	4.303	4.524	4.705	4.858	4.991	5.108	5.212	5.306	5.392	5.471	5.544
18	2.971	3.608	3.997	4.276	4.495	4.673	4.824	4.955	5.070	5.173	5.266	5.351	5.429	5.500
19	2.960	3.592	3.977	4.253	4.469	4.645	4.795	4.924	5.037	5.139	5.231	5.314	5.391	5.462
20	2.950	3.577	3.958	4.232	4.446	4.620	4.768	4.895	5.008	5.108	5.199	5.281	5.357	5.427
30	2.888	3.486	3.845	4.102	4.301	4.464	4.601	4.720	4.824	4.917	5.001	5.077	5.147	5.211

($\alpha = \mathbf{0.05}$) GOSSET'S Q-CUTOFFS, $q^*_{M,\nu;\alpha}$ $\mathbb{P}(Q > q^*_{M,\nu;\alpha}) = \alpha$

ν \ M	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
3	4.501	5.904	6.827	7.505	8.039	8.480	8.853	9.177	9.462	9.716	9.945	10.15	10.34	10.52
4	3.926	5.033	5.758	6.290	6.709	7.055	7.348	7.602	7.826	8.027	8.208	8.372	8.523	8.663
5	3.635	4.596	5.219	5.675	6.035	6.331	6.583	6.802	6.995	7.167	7.323	7.465	7.595	7.715
6	3.460	4.334	4.896	5.307	5.630	5.897	6.123	6.320	6.493	6.648	6.789	6.916	7.034	7.142
7	3.344	4.161	4.682	5.061	5.361	5.607	5.816	5.998	6.158	6.302	6.431	6.549	6.658	6.758
8	3.261	4.037	4.529	4.887	5.168	5.400	5.597	5.768	5.918	6.053	6.175	6.286	6.388	6.482
9	3.199	3.945	4.415	4.756	5.025	5.245	5.433	5.595	5.739	5.867	5.983	6.088	6.186	6.275
10	3.151	3.874	4.327	4.655	4.913	5.125	5.305	5.461	5.598	5.722	5.833	5.934	6.027	6.114
11	3.113	3.817	4.257	4.574	4.824	5.029	5.203	5.353	5.486	5.605	5.713	5.811	5.901	5.984
12	3.081	3.771	4.199	4.509	4.751	4.950	5.119	5.266	5.395	5.510	5.615	5.710	5.797	5.878
13	3.055	3.732	4.151	4.454	4.690	4.885	5.050	5.192	5.318	5.431	5.533	5.625	5.710	5.789
14	3.033	3.700	4.111	4.407	4.639	4.829	4.991	5.130	5.253	5.364	5.463	5.554	5.637	5.714
15	3.014	3.672	4.076	4.368	4.595	4.782	4.940	5.077	5.198	5.306	5.403	5.492	5.574	5.649
16	2.998	3.648	4.046	4.333	4.557	4.741	4.897	5.031	5.150	5.256	5.352	5.439	5.519	5.593
17	2.984	3.627	4.020	4.303	4.524	4.705	4.858	4.991	5.108	5.212	5.306	5.392	5.471	5.544
18	2.971	3.608	3.997	4.276	4.495	4.673	4.824	4.955	5.070	5.173	5.266	5.351	5.429	5.500
19	2.960	3.592	3.977	4.253	4.469	4.645	4.795	4.924	5.037	5.139	5.231	5.314	5.391	5.462
20	2.950	3.577	3.958	4.232	4.446	4.620	4.768	4.895	5.008	5.108	5.199	5.281	5.357	5.427
30	2.888	3.486	3.845	4.102	4.301	4.464	4.601	4.720	4.824	4.917	5.001	5.077	5.147	5.211

($\alpha = \mathbf{0.025}$) GOSSET'S Q-CUTOFFS, $q^*_{M,\nu;\alpha}$ $\mathbb{P}(Q > q^*_{M,\nu;\alpha}) = \alpha$

ν \ M	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
3	5.907	7.654	8.811	9.663	10.34	10.89	11.36	11.77	12.13	12.45	12.74	13.01	13.25	13.47
4	4.943	6.234	7.090	7.719	8.216	8.627	8.977	9.280	9.548	9.787	10.00	10.20	10.38	10.55
5	4.474	5.551	6.258	6.777	7.188	7.528	7.817	8.069	8.291	8.489	8.669	8.833	8.984	9.123
6	4.198	5.152	5.773	6.228	6.588	6.886	7.139	7.360	7.554	7.729	7.886	8.030	8.162	8.285
7	4.018	4.891	5.456	5.869	6.196	6.466	6.695	6.895	7.072	7.230	7.373	7.503	7.623	7.734
8	3.891	4.710	5.234	5.617	5.920	6.170	6.383	6.568	6.732	6.878	7.011	7.132	7.243	7.346
9	3.797	4.575	5.070	5.431	5.716	5.951	6.151	6.326	6.479	6.617	6.742	6.856	6.960	7.057
10	3.725	4.471	4.943	5.287	5.559	5.783	5.973	6.138	6.284	6.415	6.534	6.642	6.742	6.834
11	3.667	4.389	4.843	5.174	5.434	5.649	5.831	5.990	6.130	6.255	6.369	6.472	6.568	6.656
12	3.620	4.322	4.761	5.081	5.333	5.540	5.716	5.869	6.004	6.125	6.234	6.334	6.426	6.511
13	3.582	4.267	4.694	5.005	5.249	5.450	5.620	5.769	5.900	6.017	6.123	6.220	6.308	6.391
14	3.549	4.220	4.638	4.941	5.178	5.374	5.540	5.684	5.812	5.926	6.029	6.123	6.209	6.289
15	3.521	4.180	4.589	4.886	5.118	5.309	5.471	5.612	5.737	5.848	5.948	6.040	6.125	6.203
16	3.497	4.147	4.548	4.838	5.066	5.253	5.412	5.550	5.671	5.780	5.879	5.969	6.051	6.127
17	3.476	4.116	4.512	4.797	5.021	5.204	5.360	5.496	5.615	5.722	5.818	5.906	5.987	6.062
18	3.458	4.090	4.479	4.760	4.981	5.161	5.315	5.448	5.565	5.670	5.765	5.851	5.931	6.004
19	3.441	4.067	4.451	4.728	4.945	5.123	5.274	5.405	5.521	5.624	5.717	5.802	5.881	5.953
20	3.427	4.046	4.426	4.700	4.914	5.089	5.238	5.367	5.481	5.583	5.675	5.759	5.836	5.907
30	3.337	3.919	4.271	4.523	4.720	4.881	5.017	5.134	5.238	5.330	5.414	5.490	5.560	5.624

($\alpha = \mathbf{0.01}$) GOSSET'S Q-CUTOFFS, $q^*_{M,\nu;\alpha}$ $\mathbb{P}(Q > q^*_{M,\nu;\alpha}) = \alpha$

ν \ M	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
3	8.260	10.61	12.17	13.33	14.25	15.00	15.65	16.20	16.69	17.13	17.53	17.89	18.22	18.52
4	6.511	8.110	9.175	9.961	10.59	11.10	11.54	11.92	12.26	12.56	12.84	13.09	13.32	13.53
5	5.702	6.966	7.806	8.425	8.917	9.324	9.671	9.973	10.24	10.48	10.70	10.89	11.07	11.24
6	5.243	6.325	7.035	7.559	7.975	8.319	8.613	8.869	9.096	9.299	9.483	9.651	9.806	9.949
7	4.949	5.912	6.543	7.007	7.375	7.680	7.941	8.167	8.368	8.548	8.711	8.860	8.997	9.123
8	4.745	5.630	6.204	6.626	6.961	7.238	7.475	7.681	7.863	8.027	8.175	8.311	8.436	8.551
9	4.596	5.424	5.957	6.349	6.659	6.916	7.134	7.325	7.494	7.646	7.783	7.909	8.025	8.132
10	4.482	5.267	5.769	6.137	6.429	6.670	6.876	7.055	7.213	7.356	7.485	7.603	7.711	7.812
11	4.392	5.143	5.621	5.971	6.248	6.477	6.672	6.842	6.992	7.127	7.249	7.361	7.464	7.559
12	4.320	5.043	5.502	5.837	6.102	6.321	6.508	6.670	6.814	6.943	7.060	7.166	7.265	7.355
13	4.260	4.961	5.404	5.727	5.982	6.193	6.372	6.528	6.667	6.791	6.903	7.005	7.100	7.187
14	4.210	4.893	5.322	5.635	5.882	6.085	6.259	6.410	6.543	6.663	6.771	6.871	6.962	7.046
15	4.167	4.834	5.252	5.557	5.796	5.994	6.163	6.309	6.439	6.555	6.660	6.756	6.844	6.926
16	4.131	4.784	5.192	5.489	5.723	5.916	6.080	6.222	6.348	6.461	6.564	6.657	6.743	6.823
17	4.099	4.740	5.140	5.431	5.659	5.848	6.008	6.147	6.270	6.380	6.480	6.571	6.655	6.733
18	4.071	4.702	5.094	5.379	5.603	5.788	5.944	6.081	6.201	6.309	6.407	6.496	6.578	6.654
19	4.046	4.668	5.054	5.334	5.554	5.735	5.889	6.022	6.141	6.246	6.342	6.430	6.510	6.585
20	4.024	4.638	5.018	5.294	5.510	5.688	5.839	5.970	6.086	6.190	6.284	6.370	6.449	6.522
30	3.889	4.454	4.800	5.048	5.242	5.401	5.536	5.653	5.756	5.848	5.932	6.008	6.078	6.142

($\alpha = \mathbf{0.005}$) GOSSET'S Q-CUTOFFS, $q^*_{M,\nu;\alpha}$ $\mathbb{P}(Q > q^*_{M,\nu;\alpha}) = \alpha$

ν \ M	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
3	10.54	13.49	15.46	16.91	18.07	19.02	19.83	20.53	21.15	21.70	22.20	22.66	23.07	23.46
4	7.916	9.800	11.06	12.00	12.74	13.36	13.88	14.33	14.74	15.10	15.42	15.72	15.99	16.24
5	6.751	8.183	9.142	9.850	10.41	10.88	11.28	11.63	11.93	12.21	12.46	12.69	12.90	13.09
6	6.105	7.298	8.089	8.673	9.137	9.523	9.852	10.14	10.39	10.62	10.83	11.02	11.19	11.35
7	5.698	6.742	7.430	7.938	8.342	8.677	8.963	9.213	9.434	9.632	9.812	9.976	10.13	10.27
8	5.420	6.364	6.981	7.437	7.799	8.099	8.356	8.579	8.777	8.955	9.117	9.264	9.400	9.526
9	5.218	6.090	6.657	7.075	7.407	7.682	7.916	8.121	8.303	8.466	8.614	8.749	8.873	8.989
10	5.065	5.884	6.413	6.802	7.110	7.366	7.584	7.775	7.944	8.096	8.233	8.359	8.475	8.582
11	4.945	5.722	6.222	6.588	6.879	7.120	7.326	7.505	7.664	7.807	7.936	8.054	8.163	8.264
12	4.849	5.593	6.068	6.417	6.694	6.923	7.118	7.288	7.439	7.574	7.697	7.810	7.913	8.009
13	4.769	5.487	5.943	6.277	6.542	6.761	6.947	7.110	7.255	7.384	7.501	7.609	7.708	7.799
14	4.703	5.398	5.839	6.161	6.415	6.626	6.806	6.962	7.101	7.225	7.338	7.441	7.536	7.625
15	4.647	5.323	5.750	6.062	6.308	6.512	6.685	6.837	6.970	7.091	7.200	7.299	7.391	7.476
16	4.599	5.259	5.674	5.977	6.216	6.414	6.582	6.729	6.859	6.975	7.081	7.178	7.267	7.349
17	4.557	5.203	5.609	5.904	6.137	6.329	6.493	6.636	6.762	6.875	6.978	7.072	7.158	7.239
18	4.521	5.154	5.551	5.840	6.067	6.255	6.415	6.554	6.677	6.788	6.888	6.980	7.064	7.142
19	4.488	5.111	5.500	5.783	6.006	6.190	6.346	6.482	6.603	6.711	6.809	6.898	6.981	7.057
20	4.460	5.073	5.455	5.733	5.951	6.131	6.285	6.418	6.536	6.642	6.738	6.826	6.906	6.981
30	4.285	4.840	5.182	5.428	5.621	5.780	5.914	6.031	6.134	6.226	6.310	6.386	6.456	6.521

1F bcrANOVA (TUKEY POST-HOC COMPARISONS) [DEVORE 10.2]

- **SIMULTANEOUS Q-CI's:**

  Given an experiment with I groups each of size J such that the 1F bcrANOVA assumptions are satisfied.

  Then the simultaneous $100(1-\alpha)\%$ Q-CI's for all mean differences $\mu_i - \mu_j$ are:

  $$(\overline{x}_{i\bullet} - \overline{x}_{j\bullet}) \pm q^*_{I,\nu_{res};\alpha} \cdot \sqrt{\text{MS}_{err}/J} \qquad \forall i < j \qquad [\nu_{res} := I(J-1)]$$

  If Q-CI for $\mu_i - \mu_j$ does not contain zero, then μ_i & μ_j significantly differ.

- **TUKEY COMPLETE PAIRWISE POST-HOC COMPARISON:** (Simpler than finding simultaneous Q-CI's)

  Given an experiment with I groups each of size J where 1F bcrANOVA rejects H_0^A at significance level α.

  Then, to determine which population means significantly differ:

  1. Compute significant difference width $w = q^*_{I,\nu_{res};\alpha} \cdot \sqrt{\text{MS}_{res}/J}$ $\quad [\nu_{res} := I(J-1)]$
  2. Sort the group means in ascending order: $\overline{x}_{(1)\bullet} \leq \overline{x}_{(2)\bullet} \leq \cdots \leq \overline{x}_{(I)\bullet}$
  3. For each sorted group mean $\overline{x}_{(k)\bullet}$:
     - If $\overline{x}_{(k+1)\bullet} \notin \left[\overline{x}_{(k)\bullet},\ \overline{x}_{(k)\bullet} + w\right]$, repeat STEP 3 with next sorted group mean.
     - Else, underline $\overline{x}_{(k)\bullet}$ and all larger means within a distance of w with new line.

  Interpretation of underlining:

  - Group means sharing a common underline implies they are not significantly different from one another.
  - Group means not sharing a common underline implies they are significantly different from one another.

- **t-CI's FOR COMPARING COLLECTIONS OF GROUP MEANS:**

  Given an experiment with I groups each of size J such that the 1F bcrANOVA assumptions are satisfied.

  Let constants $c_1, c_2, \cdots, c_I \in \mathbb{R}$ such that they sum to zero: $\sum_i c_i = 0$ $\quad [\nu_{res} := I(J-1)]$

  Then the $100(1-\alpha)\%$ t-CI for mean collection difference $\sum_i c_i\mu_i$ is: $\left(\sum_i c_i \overline{x}_{i\bullet}\right) \pm t^*_{\nu_{res};\alpha/2} \cdot \sqrt{\text{MS}_{res} \cdot \sum_i c_i^2/J}$

- **EXAMPLES OF COMPARING COLLECTIONS OF GROUP MEANS:**

# GROUPS:	COLLECTIONS TO COMPARE:	$\sum_i c_i\mu_i$
$I=3$	μ_1 vs. (μ_2, μ_3)	$\mu_1 - \frac{1}{2}(\mu_2+\mu_3)$
$I=3$	(μ_1, μ_3) vs. μ_2	$\frac{1}{2}(\mu_1+\mu_3) - \mu_2$
$I=4$	μ_1 vs. (μ_2, μ_3, μ_4)	$\mu_1 - \frac{1}{3}(\mu_2+\mu_3+\mu_4)$
$I=4$	(μ_1, μ_2) vs. (μ_3, μ_4)	$\frac{1}{2}(\mu_1+\mu_2) - \frac{1}{2}(\mu_3+\mu_4)$

- **OTHER POST-HOC & PLANNED COMPARISONS♠:**

METHOD:	YEAR:	WORKS FOR UNEQUAL SIZES?	WORKS FOR UNEQUAL VARIANCES?	REMARKS:
Bonferroni-Dunn	1958	NO	NO	Very conservative t-tests with correction to α-levels
Tukey	1953	NO	NO	Ideal for all-pairwise post-hoc, robust to non-normality
Tukey-Kramer	1956	YES	NO	
Fisher	1949	YES	NO	Sequential pairwise post-hoc, but for $I > 3, \alpha_{exp}$ grows!
Fisher-Hayter	1986	YES	NO	Fixes α_{exp} defect in Fisher, more powerful than Tukey
Scheffé	1953	YES	NO	Conservative but flexible, ideal for complex post-hoc
Kaiser-Bowden	1983	YES	YES	
Dunnett	1955	YES	NO	Planned comparisons of groups to a control group
Dunnett T3	1980	YES	YES	Ideal for unequal variance case when $J < 50$
Dunnett C	1980	YES	YES	Ideal for unequal variance case when $J \geq 50$

Besides Tukey & Tukey-Kramer, no other post-hoc or planned comparisons will be considered in this course.

♠First textbook to survey many of these multiple comparisons: R.G. Miller, *Simultaneous Statistical Inference*, Springer, 1966.

EX 10.2.1: Dentists use resin composites and ceramic fillings among others for cavities in teeth. The shear bond strengths of resin composite-ceramic bonds formed from three possible configurations (conventional, all-composite, reversed) were measured (in MPa) and summarized in the following table:

GROUP:	**SAMPLE SIZE:**	**MEAN:**	**STD DEV:**
Conventional	11	$\overline{x}_{1\bullet} = 10.37$	$s_1 = 1.99$
All-Composite	11	$\overline{x}_{2\bullet} = 20.12$	$s_2 = 2.45$
Reversed	11	$\overline{x}_{3\bullet} = 18.02$	$s_3 = 2.52$

A similar table and all the details regarding the experiment can be found in the following paper:

A. Della Bona, R. van Noort, "Shear vs. Tensile Bond Strength of Resin Composite Bonded to Ceramic", *Journal of Dental Research*, **74** (1995), 1591-1596.

A 1-Factor ANOVA at significance level $\alpha = 0.05$ was performed, resulting in the rejection of the null hypothesis.

The error mean square was found to be $\text{MS}_{res} \approx 5.4377$ during the ANOVA procedure.

(a) Perform the Tukey Complete Pairwise Post-Hoc Comparison to determine which groups significantly differ.

(b) Compute the 95% t-CI comparing the conventional config to the reversed and all-composite configurations.