**EX 12.3.1:** The systolic blood pressures of twelve adults were measured as a single group without intervention†:

| SYSTOLIC BLOOD PRESSURE (in mmHg) versus Age (in years) | | | | | | | | | | | | | TOTAL |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Index ($i$) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | $n = 12$ |
| Age ($x_i$) | 40 | 41 | 43 | 44 | 44 | 46 | 48 | 50 | 52 | 54 | 54 | 55 | $\sum_i x_i = 571$ |
| BP ($y_i$) | 158.8 | 158.5 | 163.1 | 165.1 | 166.3 | 165.7 | 172.1 | 174.5 | 173.8 | 176.2 | 171.2 | 177.2 | $\sum_i y_i = 2022.5$ |

†(Simplification of data) Y.H. Chan, "Biostatistics 201: Linear Regression Analysis", *Singapore Med. J.*, **45** (2004), 55-61.

(a) Compute all sums: $S_x, S_y, S_{xx}, S_{yy}, S_{xy}$.

$S_x := \sum_i x_i \overset{TABLE}{=} 571 \implies \boxed{S_x = 571}$ $\quad S_y := \sum_i y_i \overset{TABLE}{=} 2022.5 \implies \boxed{S_y = 2022.5}$

$S_{xx} := \sum_i x_i x_i = 27483 \implies \boxed{S_{xx} = 27483}$ $\quad S_{yy} := \sum_i y_i y_i = 341342.31 \implies \boxed{S_{yy} = 341342.31}$

$S_{xy} := \sum_i x_i y_i = 96596.6 \implies \boxed{S_{xy} = 96596.6}$

(b) Compute all centered sums: $SC_{xx}, SC_{yy}, SC_{xy}$.

$\boxed{SC_{xx} = S_{xx} - \frac{1}{n} S_x S_x \approx 312.916666667}$ $\boxed{SC_{yy} = S_{yy} - \frac{1}{n} S_y S_y \approx 466.789166667}$ $\boxed{SC_{xy} = S_{xy} - \frac{1}{n} S_x S_y \approx 359.308333333}$

(c) Compute the ordinary least-squares (OLS) estimates of the regression parameters, $\hat{\beta}_0, \hat{\beta}_1$.

$\hat{\beta}_1 \overset{OLS}{=} \frac{SC_{xy}}{SC_{xx}} \approx 1.148255659 \implies \boxed{\hat{\beta}_1 \approx 1.148}$ $\quad \hat{\beta}_0 \overset{OLS}{=} \frac{S_y - \hat{\beta}_1 S_x}{n} \approx 113.903834893 \implies \boxed{\hat{\beta}_0 \approx 113.904}$

(d) Compute the residual sum of squares, $\text{SS}_{res}$.

$\text{SS}_{res} = SC_{yy} - \hat{\beta}_1^2 \cdot SC_{xx} = 466.789166667 - (1.148255659)^2 \cdot (312.916666667) \approx 54.211339634 \implies \boxed{\text{SS}_{res} \approx 54.211339634}$

(e) Compute the linear model's estimated variance, $\hat{\sigma}^2$.

$\hat{\sigma}^2 = \frac{\text{SS}_{res}}{n-2} = \frac{54.211339634}{12-2} \approx 5.421133963 \implies \boxed{\hat{\sigma}^2 \approx 5.421}$

(f) Compute the estimated variance of estimated OLS slope parameter, $\hat{\mathbb{V}}[\hat{\beta}_1]$.

$\hat{\mathbb{V}}[\hat{\beta}_1] = \frac{\hat{\sigma}^2}{SC_{xx}} = \frac{5.421133963}{312.916666667} \approx 0.017324529 \implies \boxed{\hat{\mathbb{V}}[\hat{\beta}_1] \approx 0.017}$

(g) Compute the estimated standard deviation of estimated OLS slope parameter, $\hat{\mathbb{D}}[\hat{\beta}_1]$.

$\hat{\mathbb{D}}[\hat{\beta}_1] = \sqrt{\hat{\mathbb{V}}[\hat{\beta}_1]} \approx \sqrt{0.017324529} \approx 0.131622677 \implies \boxed{\hat{\mathbb{D}}[\hat{\beta}_1] \approx 0.132}$

(h) Construct a 95% $t$-CI for slope parameter $\beta_1$.

$\nu_{res} = n - 2 = 12 - 2 = 10 \implies t^*_{\nu_{res};\alpha/2} = t^*_{10;0.025} \overset{LOOKUP}{\approx} 2.228$

$\therefore$ 95% $t$-CI: $\quad \hat{\beta}_1 \pm t^*_{\nu_{res};\alpha/2} \cdot \hat{\mathbb{D}}[\hat{\beta}_1] \iff 1.148 \pm 2.228 \cdot 0.132 \iff \boxed{(\mathbf{0.854},\ \mathbf{1.442})}$

(i) Perform a model utility $F$-test of slope parameter $\beta_1$ with $\alpha = 0.05$ significance level. Also, build its ANOVA table.

$\boxed{H_0 : \beta_1 = 0 \text{ vs. } H_A : \beta_1 \neq 0}$

$$\begin{array}{lcccccccc} \text{SS}_{total} & = & SC_{yy} & \approx & 466.789166667 & \implies & \nu_{total} = n-1 = 12-1 = 11 \\ \text{SS}_{res} & = & SC_{yy} - \hat{\beta}_1^2 \cdot SC_{xx} & \approx & 54.211339634 & \implies & \nu_{res} = n-2 = 12-2 = 10 \\ \text{SS}_{reg} & = & \hat{\beta}_1^2 \cdot SC_{xx} & \approx & 412.577827033 & \implies & \nu_{reg} = 1 \end{array}$$

$\text{MS}_{res} := \frac{\text{SS}_{res}}{\nu_{res}} = \frac{54.211339634}{10} \approx 5.421133963, \quad \text{MS}_{reg} := \frac{\text{SS}_{reg}}{\nu_{reg}} = \frac{412.577827033}{1} \approx 412.577827033$

$\implies f_{reg} = \frac{\text{MS}_{reg}}{\text{MS}_{res}} = \frac{412.577827033}{5.421133963} \approx 76.105447652 \implies \boxed{f_{reg} \approx 76.105}$

$\implies p_{reg} = 1 - \Phi_F(f_{reg}; \nu_{reg}, \nu_{res}) \approx 1 - 0.9999945285388567 \approx 5.471461143269352 \times 10^{-6} \implies \boxed{p_{reg} \approx 5.471 \times 10^{-6}}$

$\therefore\ p_{reg} \approx 5.471 \times 10^{-6} \leq 0.05 = \alpha \implies$ **Reject $H_0$**. There's enough evidence from this obs. study to claim $\beta_1 \neq 0$.

**Model Utility $F$-Test (Significance Level $\alpha = 0.05$)**

| Variation Source | df | Sum of Squares | Mean Square | $F$ Stat Value | P-value | Decision |
|---|---|---|---|---|---|---|
| Regression | 1 | 412.578 | 412.578 | 76.105 | $5.5 \times 10^{-6}$ | Reject $H_0$ |
| Unknown | 10 | 54.211 | 5.421 | | | |
| Total | 11 | 466.789 | | | | |