EX 9.1.1: A manufacturing plant produces a certain type of brake pad used in big rig trucks.

A sample of 55 day-shift workers showed that the mean # of brake pads produced was 310 with a std dev of 20.

A sample of 60 night-shift workers showed that the mean # of brake pads produced was 325 with a std dev of 26.

Assume that the two samples are independent.

Do these samples suggest that the average # brake pads produced by the two shifts differ?

(Use significance level $\alpha = 0.05$)

(a) State the appropriate null hypothesis H_0 & alternative hypothesis H_A.

Let $\mu_1 \equiv$ (Mean # of brake pads produced by day shift)
Let $\mu_2 \equiv$ (Mean # of brake pads produced by night shift)

$$\Longrightarrow \begin{array}{ll} H_0: & \mu_1 = \mu_2 \\ H_A: & \mu_1 \neq \mu_2 \end{array} \Longrightarrow \boxed{\begin{array}{ll} H_0: & \mu_1 - \mu_2 = 0 \\ H_A: & \mu_1 - \mu_2 \neq 0 \end{array}}$$

(b) Compute the appropriate z-test statistic value for this hypothesis test.

$$z = \frac{(\overline{x} - \overline{y}) - \delta_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(310 - 325) - 0}{\sqrt{\frac{20^2}{55} + \frac{26^2}{60}}} \approx \boxed{\mathbf{-3.48372}}$$

(c) Identify the two appropriate z-cutoffs and lookup/compute their values.

$$z^*_{\alpha/2} = z^*_{0.025} \overset{LOOKUP}{\approx} \boxed{\mathbf{1.960}} \qquad z^*_{1-\alpha/2} \overset{SYM}{=} -z^*_{\alpha/2} = -z^*_{0.025} \approx \boxed{\mathbf{-1.960}}$$

(d) Compute the appropriate P-value using software.

$$\text{P-value} = 2 \cdot [1 - \Phi(|z|)] = 2 \cdot [1 - \Phi(3.48372)] \overset{SW}{\approx} 2 \cdot [1 - 0.9997528] = \boxed{\mathbf{0.0004944}}$$

(e) Using the computed z-cutoffs, render the appropriate decision.

Since $z < 0$ and $z < z^*_{1-\alpha/2}$, $\boxed{\textbf{Reject } H_0 \textbf{ in favor of } H_A}$

(f) Using the computed P-value, render the appropriate decision.

Since P-value $\approx 0.0004944 < 0.05 = \alpha$, $\boxed{\textbf{Reject } H_0 \textbf{ in favor of } H_A}$

The sample evidence is compelling enough to conclude that it's plausible that the avg # of brake pads produced by the two shifts differ.

(g) Construct the approximate 95% z-CI for $\mu_1 - \mu_2$.

$$(\overline{x} - \overline{y}) \pm z^*_{\alpha/2} \cdot \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \Longrightarrow (310 - 325) \pm \mathbf{1.960} \cdot \sqrt{\frac{20^2}{55} + \frac{26^2}{60}} \Longrightarrow -15 \pm 8.4392 \Longrightarrow \boxed{(\mathbf{-23.4392},\ \mathbf{-6.5608})}$$

A negative sign in the CI indicates that the night shift produces more brake pads on avg than the day shift.

Notice that the CI does not contain zero, which is expected since H_0 was rejected.