STANDARD NORMAL DISTRIBUTION [DEVORE 9.1]

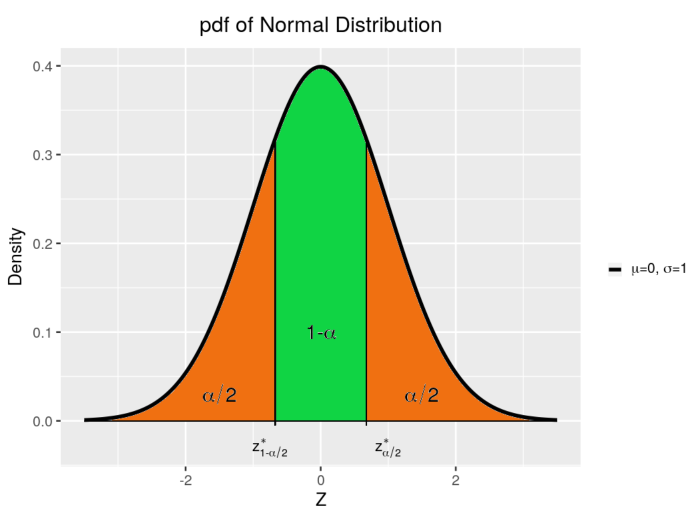
- **STANDARD NORMAL DISTRIBUTION (DEFINITION):**

Notation	$Z \sim \text{Normal}(0,1)$ or $Z \sim \text{StdNormal}$
Parameters	(None)
Support	$\text{Supp}(Z) = (-\infty, \infty)$
pdf	$f_Z(z) := \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$
cdf	$\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\xi^2/2}\, d\xi$
Mean	$\mathbb{E}[Z] = 0$
Variance	$\mathbb{V}[Z] = 1$
Model(s)	(Used mainly for Statistical Inference)

- **STANDARD NORMAL DISTRIBUTION (PROPERTIES):**
  - The standard normal pdf curve is unimodal.
  - The standard normal pdf curve is bell-shaped.
  - The standard normal pdf curve is symmetric.

- **z-CUTOFFS (AKA z CRITICAL VALUES):**
  - Value z^*_α is a **z-cutoff (AKA z critical value)** of std normal distribution such that $\mathbb{P}(Z > z^*_\alpha) = \alpha$
  - NOTE: Do not confuse z-cutoff z^*_α with z percentile z_α: $\mathbb{P}(Z \leq z_\alpha) = \alpha$
  - Lower-tail z-cutoffs can be found from upper-tail z-cutoffs: $z^*_{1-\alpha} = -z^*_\alpha$ (this follows since std normal is symmetric)

- **STANDARD NORMAL DISTRIBUTION (EXAMPLE PDF PLOT):**



STD NORMAL z-CUTOFFS, z^*_α $\quad \mathbb{P}(Z > z^*_\alpha) = \alpha, \quad z^*_{1-\alpha} = -z^*_\alpha$

α	**0.2**	**0.1**	**0.05**	**0.025**	**0.02**	**0.01**	**0.005**	**0.001**	**0.0005**
z^*_α	0.842	1.282	1.645	1.960	2.054	2.326	2.576	3.090	3.291

LARGE-SAMPLE z-TESTS & z-CI'S FOR $\mu_1 - \mu_2$ [DEVORE 9.1]

- **A STATISTIC RELATED TO THE STANDARD NORMAL DISTRIBUTION:**

  Given any two populations with means μ_1, μ_2 and unknown σ_1, σ_2.

  Let $\mathbf{X} := (X_1, \ldots, X_{n_1})$ & $\mathbf{Y} := (Y_1, \ldots, Y_{n_2})$ be random samples from the 1^{st} & 2^{nd} populations, respectively.

  Moreover, suppose random samples $\mathbf{X}$ & $\mathbf{Y}$ are independent of each other and their sizes are "large", meaning $n_1, n_2 > 40$.

  Then the following statistic has an approximate std normal distribution: $\dfrac{(\overline{X} - \overline{Y}) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{S_1^2}{n_1} + \dfrac{S_2^2}{n_2}}} \overset{approx}{\sim}$ StdNormal

- **LARGE-SAMPLE z-TESTS FOR $\mu_1 - \mu_2$:**

Population:	Any Two Populations with unknown σ_1, σ_2
Realized Samples:	$\mathbf{x} := (x_1, x_2, \cdots, x_{n_1})$, mean $\overline{x}$, std dev s_1 $(n_1 > 40)$ $\mathbf{y} := (y_1, y_2, \cdots, y_{n_2})$, mean $\overline{y}$, std dev s_2 $(n_2 > 40)$ Samples $\mathbf{x}$ & $\mathbf{y}$ are independent of each other
Test Statistic Value: $W(\mathbf{x}, \mathbf{y}; \delta_0)$	$z = \dfrac{(\overline{x} - \overline{y}) - \delta_0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$

HYPOTHESIS TEST:	REJECTION REGION AT LVL α:	P-VALUE DETERMINATION:
$H_0 : \mu_1 - \mu_2 = \delta_0$ vs. $H_A : \mu_1 - \mu_2 > \delta_0$	$z \geq z^*_\alpha$	P-value $= 1 - \Phi(z)$
$H_0 : \mu_1 - \mu_2 = \delta_0$ vs. $H_A : \mu_1 - \mu_2 < \delta_0$	$z \leq z^*_{1-\alpha}$	P-value $= \Phi(z)$
$H_0 : \mu_1 - \mu_2 = \delta_0$ vs. $H_A : \mu_1 - \mu_2 \neq \delta_0$	$z \leq z^*_{1-\alpha/2}$ or $z \geq z^*_{\alpha/2}$	P-value $= 2 \cdot [1 - \Phi(\lvert z \rvert)]$

DECISION RULE: If P-value $\leq \alpha$ then reject H_0 in favor of H_A
If P-value $> \alpha$ then accept H_0 (i.e. fail to reject H_0)

- **LARGE-SAMPLE z-CI's FOR $\mu_1 - \mu_2$:**

  Given any two populations with means μ_1, μ_2 and unknown σ_1, σ_2.

  Let $\mathbf{x} := (x_1, x_2, \cdots, x_{n_1})$ be a sample taken from the 1^{st} population with $n_1 > 40$.

  Let $\mathbf{y} := (y_1, y_2, \cdots, y_{n_2})$ be a sample taken from the 2^{nd} population with $n_2 > 40$.

  Moreover, suppose samples $\mathbf{x}$ & $\mathbf{y}$ are independent of each other.

  Then the $100(1-\alpha)\%$ **large-sample z-CI for $\mu_1 - \mu_2$** is

$$\left((\overline{x} - \overline{y}) - z^*_{\alpha/2} \cdot \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}},\ \ (\overline{x} - \overline{y}) + z^*_{\alpha/2} \cdot \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \right)$$

—- OR WRITTEN MORE COMPACTLY —-

$$(\overline{x} - \overline{y}) \pm z^*_{\alpha/2} \cdot \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

EX 9.1.1: A manufacturing plant produces a certain type of brake pad used in big rig trucks.

A sample of 55 day-shift workers showed that the mean # of brake pads produced was 310 with a std dev of 20.

A sample of 60 night-shift workers showed that the mean # of brake pads produced was 325 with a std dev of 26.

Assume that the two samples are independent.

Do these samples suggest that the average # brake pads produced by the two shifts differ?

(Use significance level $\alpha = 0.05$)

(a) State the appropriate null hypothesis H_0 & alternative hypothesis H_A.

(b) Compute the appropriate z-test statistic value for this hypothesis test.

(c) Identify the two appropriate z-cutoffs and lookup/compute their values.

(d) Compute the appropriate P-value using software.

(e) Using the computed z-cutoffs, render the appropriate decision.

(f) Using the computed P-value, render the appropriate decision.

(g) Construct the approximate 95% z-CI for $\mu_1 - \mu_2$.