EX 9.2.1: A manufacturing plant produces a certain type of brake pad used in big rig trucks.

A sample of 10 day-shift workers showed that the mean # of brake pads produced was 52 with a std dev of 4.4.

A sample of 13 night-shift workers showed that the mean # of brake pads produced was 55 with a std dev of 4.6.

Assume that the two samples are independent and the two corresponding populations are normally distributed.

Do these samples suggest that the average # brake pads produced by the two shifts differ?

(Use significance level $\alpha = 0.01$)

(a) State the appropriate null hypothesis H_0 & alternative hypothesis H_A.

Let $\mu_1 \equiv$ (Mean # of brake pads produced by day shift)
Let $\mu_2 \equiv$ (Mean # of brake pads produced by night shift)
$\implies \begin{array}{ll} H_0: & \mu_1 = \mu_2 \\ H_A: & \mu_1 \neq \mu_2 \end{array} \implies \boxed{\begin{array}{ll} H_0: & \mu_1 - \mu_2 = 0 \\ H_A: & \mu_1 - \mu_2 \neq 0 \end{array}}$

(b) Compute the independent t-test statistic value for this hypothesis test.

$$t = \frac{(\overline{x} - \overline{y}) - \delta_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(52 - 55) - 0}{\sqrt{\frac{4.4^2}{10} + \frac{4.6^2}{13}}} \approx \boxed{\mathbf{-1.58917}}$$

(c) Identify the two appropriate independent t-cutoffs and lookup/compute their values.

First, determine the appropriate # degrees of freedom:

$$\nu^* = \left\lfloor \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{(s_1^2/n_1)^2}{n_1 - 1} + \frac{(s_2^2/n_2)^2}{n_2 - 1}} \right\rfloor = \left\lfloor \frac{\left(\frac{4.4^2}{10} + \frac{4.6^2}{13}\right)^2}{\frac{(4.4^2/10)^2}{10 - 1} + \frac{(4.6^2/13)^2}{13 - 1}} \right\rfloor \approx \left\lfloor \frac{12.69990286}{0.637236965} \right\rfloor = \lfloor 19.92963929 \rfloor = 19$$

$$t^*_{\nu^*;\alpha/2} = t^*_{19;0.005} \overset{LOOKUP}{\approx} \boxed{\mathbf{2.861}} \qquad t^*_{\nu^*;1-\alpha/2} \overset{SYM}{=} -t^*_{\nu^*;\alpha/2} = -t^*_{19;0.005} \approx \boxed{\mathbf{-2.861}}$$

(d) Compute the appropriate P-value using software.

$$\text{P-value} = 2 \cdot [1 - \Phi_t(|t|; \nu^*)] = 2 \cdot [1 - \Phi_t(1.58917; \nu = 19)] \overset{SW}{\approx} 2 \cdot [1 - 0.9357] = \boxed{\mathbf{0.1286}}$$

(e) Using the computed independent t-cutoffs, render the appropriate decision.

Since $t^*_{\nu^*;1-\alpha/2} < t < t^*_{\nu^*;\alpha/2}$, $\boxed{\textbf{Accept (or Fail to Reject) } H_0}$

(f) Using the computed P-value, render the appropriate decision.

Since P-value $\approx 0.1286 > 0.01 = \alpha$, $\boxed{\textbf{Accept (or Fail to Reject) } H_0}$

There is not enough compelling evidence from the data to conclude
that the avg # of brake pads produced by the two shifts differ.

(g) Construct the approximate 99% independent t-CI for $\mu_1 - \mu_2$.

$$(\overline{x} - \overline{y}) \pm t^*_{\nu^*;\alpha/2} \cdot \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \implies (52 - 55) \pm \mathbf{2.861} \cdot \sqrt{\frac{4.4^2}{10} + \frac{4.6^2}{13}} \implies -3 \pm 5.4009 \implies \boxed{(\mathbf{-8.4009},\ \mathbf{2.4009})}$$

A negative sign in the CI indicates that the night shift produces more brake pads on avg than the day shift.
A positive sign in the CI indicates that the day shift produces more brake pads on avg than the night shift.

Notice that the CI contains zero, which is expected since H_0 was accepted.

EX 9.2.2: A manufacturing plant produces a certain type of brake pad used in big rig trucks.

A sample of 10 day-shift workers showed that the mean # of brake pads produced was 52 with a std dev of 4.4.

A sample of 13 night-shift workers showed that the mean # of brake pads produced was 55 with a std dev of 4.6.

Assume that the two samples are independent and the two corresponding populations are normally distributed.

Do these samples suggest that the average # brake pads produced by the two shifts differ?

(Use significance level $\alpha = 0.01$)

(a) State the appropriate null hypothesis H_0 & alternative hypothesis H_A.

$$\begin{array}{l}\text{Let } \mu_1 \equiv (\text{Mean \# of brake pads produced by day shift}) \\ \text{Let } \mu_2 \equiv (\text{Mean \# of brake pads produced by night shift})\end{array} \implies \begin{array}{ll} H_0: & \mu_1 = \mu_2 \\ H_A: & \mu_1 \neq \mu_2 \end{array} \implies \boxed{\begin{array}{ll} H_0: & \mu_1 - \mu_2 = 0 \\ H_A: & \mu_1 - \mu_2 \neq 0 \end{array}}$$

(b) Compute the pooled t-test statistic value for this hypothesis test.

1^{st}, compute pooled sample variance: $s^2_{pool} = \dfrac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2} = \dfrac{(10-1)\cdot 4.4^2 + (13-1)\cdot 4.6^2}{10+13-2} \approx 20.38857$

$$t_{pool} = \frac{(\overline{x}-\overline{y})-\delta_0}{\sqrt{s^2_{pool}\cdot\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{(52-55)-0}{\sqrt{20.38857\cdot\left(\frac{1}{10}+\frac{1}{13}\right)}} \approx \boxed{\mathbf{-1.5796}}$$

(c) Identify the two appropriate pooled t-cutoffs and lookup/compute their values.

$\nu_{pool} = n_1 + n_2 - 2 = 10 + 13 - 2 = 21$

$t^*_{\nu_{pool};\alpha/2} = t^*_{21;0.005} \overset{LOOKUP}{\approx} \boxed{\mathbf{2.831}}$ $\qquad t^*_{\nu_{pool};1-\alpha/2} = -t^*_{\nu_{pool};\alpha/2} \overset{LOOKUP}{\approx} \boxed{\mathbf{-2.831}}$

(d) Compute the appropriate P-value using software.

$$\text{P-value} = 2\cdot[1-\Phi_t(|t_{pool}|;\nu_{pool})] = 2\cdot[1-\Phi_t(1.5796;\nu=21)] \overset{SW}{\approx} 2\cdot[1-0.9354] = \boxed{\mathbf{0.1292}}$$

(e) Using the computed pooled t-cutoffs, render the appropriate decision.

Since $t^*_{\nu_{pool};1-\alpha/2} < t_{pool} < t^*_{\nu_{pool};\alpha/2}$, $\boxed{\textbf{Accept (or Fail to Reject) } H_0}$

(f) Using the computed P-value, render the appropriate decision.

Since P-value $\approx 0.1292 > 0.01 = \alpha$, $\boxed{\textbf{Accept (or Fail to Reject) } H_0}$

There is not enough compelling evidence from the data to conclude
that the avg # of brake pads produced by the two shifts differ.

(g) Construct the 99% pooled t-CI for $\mu_1 - \mu_2$.

$$(\overline{x}-\overline{y}) \pm t^*_{\nu_{pool};\alpha/2}\cdot\sqrt{s^2_{pool}\cdot\left(\frac{1}{n_1}+\frac{1}{n_2}\right)} \implies (52-55) \pm \mathbf{2.831}\cdot\sqrt{20.38857\cdot\left(\frac{1}{10}+\frac{1}{13}\right)}$$

$$\implies -3 \pm 5.3768 \implies \boxed{(\mathbf{-8.3768},\ \mathbf{2.3768})}$$

A negative sign in the CI indicates that the night shift produces more brake pads on avg than the day shift.
A positive sign in the CI indicates that the day shift produces more brake pads on avg than the night shift.

Notice that the CI contains zero, which is expected since H_0 was accepted.