EX 9.3.1: The lifetimes of two light bulb brands were measured via two dependent samples and summarized into this table:

BULB BRAND	(BULB LIFETIMES in yrs)
Brand #1 (x)	9.22, 9.07, 8.95, 8.98, 9.54
Brand #2 (y)	8.92, 8.88, 9.10, 8.71, 8.85

Do these samples suggest that the average bulb lifetimes differ by brand? (Use significance level $\alpha = 0.01$)

(a) Define μ_1, μ_2 and state the appropriate null hypothesis H_0 & alternative hypothesis H_A in terms of μ_1, μ_2.

Let: $\begin{cases} \mu_1 \equiv \text{Average lifetime of all brand \#1 light bulbs} \\ \mu_2 \equiv \text{Average lifetime of all brand \#2 light bulbs} \end{cases} \implies \boxed{\begin{array}{ll} H_0: & \mu_1 = \mu_2 \\ H_A: & \mu_1 \neq \mu_2 \end{array}}$

(b) Define r.v. D, sample $\mathbf{d}$ and parameter μ_D. Then state the appropriate hypotheses in terms of μ_D.

$$D := X - Y \implies D \sim \text{Normal}(\mu_D, \sigma_D^2) \implies \mu_D = \mu_1 - \mu_2$$

$$\begin{aligned} \mathbf{d} \equiv (d_1, d_2, d_3, d_4, d_5) &:= (x_1 - y_1, x_2 - y_2, x_3 - y_3, x_4 - y_4, x_5 - y_5) \\ &= (9.22 - 8.92,\ 9.07 - 8.88,\ 8.95 - 9.10,\ 8.98 - 8.71,\ 9.54 - 8.85) \\ &= (0.30,\ 0.19,\ -0.15,\ 0.27,\ 0.69) \end{aligned}$$

$\therefore \boxed{\mathbf{d} = (0.30,\ 0.19,\ -0.15,\ 0.27,\ 0.69)}$ $\quad \boxed{\begin{array}{ll} H_0: & \mu_D = 0 \\ H_A: & \mu_D \neq 0 \end{array}}$

(c) Compute the paired t-test statistic value for the hypothesis test concerning μ_D.

1st, compute realized differences' mean & std dev:

$$\begin{array}{llllll} \text{Mean} & \overline{d} := & \frac{1}{n}\sum_{k=1}^{n} d_k & = & \frac{1}{5}\left[0.30 + 0.19 + (-0.15) + 0.27 + 0.69\right] & = 0.26 \\ \text{Variance} & s_d^2 := & \frac{1}{n-1}\sum_{k=1}^{n}(d_k - \overline{d})^2 & = & \frac{1}{4}\left[\begin{array}{l} (0.30 - 0.26)^2 + (0.19 - 0.26)^2 + (-0.15 - 0.26)^2 \\ +(0.27 - 0.26)^2 + (0.69 - 0.26)^2 \end{array}\right] & = 0.0899 \\ \text{Std Dev} & s_d := & \sqrt{s_d^2} & = & \sqrt{0.0899} & \approx 0.2998 \end{array}$$

2nd, compute paired t-test statistic value:

$$t = \frac{\overline{d} - \delta_0}{s_d/\sqrt{n}} = \frac{0.26 - 0}{0.2998/\sqrt{5}} \approx \boxed{\mathbf{1.9392}}$$

(d) Identify the two appropriate paired t-cutoffs and lookup/compute their values.

$$t^*_{n-1;\alpha/2} = t^*_{4;0.005} \overset{LOOKUP}{\approx} \boxed{\mathbf{4.604}} \qquad t^*_{n-1;1-\alpha/2} \overset{SYM}{=} -t^*_{n-1;\alpha/2} = -t^*_{4;0.005} \approx \boxed{\mathbf{-4.604}}$$

(e) Compute the appropriate P-value using software.

$$\text{P-value} = 2 \cdot [1 - \Phi_t(|t|; \nu = n-1)] = 2 \cdot [1 - \Phi_t(1.9392; \nu = 4)] \overset{SW}{\approx} 2 \cdot [1 - 0.9378] = \boxed{\mathbf{0.1244}}$$

(f) Using the computed paired t-cutoffs, render the appropriate decision.

Since $t^*_{n-1;1-\alpha/2} < t < t^*_{n-1;\alpha/2}$, $\boxed{\textbf{Accept (or Fail to Reject) } H_0}$

(g) Using the computed P-value, render the appropriate decision.

Since P-value $\approx 0.1244 > 0.01 = \alpha$, $\boxed{\textbf{Accept (or Fail to Reject) } H_0}$

There is not enough compelling evidence from the data to support the claim that the average lifetimes of Brand #1 light bulbs & Brand #2 light bulbs differ.

(h) Construct the 99% paired t-CI for μ_D.

$$\overline{d} \pm t^*_{n-1;\alpha/2} \cdot \frac{s_d}{\sqrt{n}} \implies 0.26 \pm t^*_{4;0.005} \cdot \frac{0.2998}{\sqrt{5}} \overset{LOOKUP}{\implies} 0.26 \pm \mathbf{4.604} \cdot \frac{0.2998}{\sqrt{5}} \implies 0.26 \pm 0.6173$$

$\therefore$ 99% paired t-CI for μ_D is $\boxed{(\mathbf{-0.3573},\ \mathbf{0.8773})}$ Notice CI contains zero, which is expected since H_0 was accepted.

(i) Identify some possible nuisance factors for this experiment.

One brand of bulbs may have been manufactured from a different raw material batch than the other brand.
One brand of bulbs may have been manufactured by a different machine/factory than the other brand.
Both brands of bulbs may have same raw material batch & machine, but different machine operators.
Same batch, machine and operator – but all Brand # 1 bulbs were made first. Operator fatigue may have set in.

EX 9.3.2: Dentists use resin composites and ceramic fillings among others for cavities in teeth. The shear bond strengths of resin composite-ceramic bonds formed from two possible configurations (conventional & all-composite) were measured (in MPa) via two dependent samples and summarized in the following table:

GROUP:	SAMPLE SIZE:	MEAN:	STD DEV:	COVARIANCE:
Conventional (x)	10	$\overline{x} = 10.62$	$s_x = 2.3300$	$s_{xy} = -2.2447$
All-Composite (y)	10	$\overline{y} = 21.71$	$s_y = 2.0684$	

This simplified table and all the details regarding the experiment can be found in the following paper:

A. Della Bona, R. van Noort, "Shear vs. Tensile Bond Strength of Resin Composite Bonded to Ceramic", *Journal of Dental Research*, **74** (1995), 1591-1596.

Does this experiment suggest that the average shear bond strengths of resin bonds differ by configuration?

(Use significance level $\alpha = 0.05$)

(a) Define μ_1, μ_2 and state the appropriate null hypothesis H_0 & alternative hypothesis H_A in terms of μ_1, μ_2.

Let: $\begin{cases} \mu_1 \equiv \text{Average shear bond strength of all conventionally-formed resin bonds} \\ \mu_2 \equiv \text{Average shear bond strength of all all-composite-formed resin bonds} \end{cases} \implies \boxed{\begin{array}{ll} H_0: & \mu_1 = \mu_2 \\ H_A: & \mu_1 \neq \mu_2 \end{array}}$

(b) Define r.v. D and parameter μ_D. Then state the appropriate hypotheses in terms of μ_D.

$D := X - Y \implies D \sim \text{Normal}(\mu_D, \sigma_D^2) \implies \mu_D = \mu_1 - \mu_2$

$\therefore \boxed{\begin{array}{ll} H_0: & \mu_D = 0 \\ H_A: & \mu_D \neq 0 \end{array}}$

(c) Compute the paired t-test statistic value for the hypothesis test concerning μ_D.

1[st], compute realized differences' mean & std dev:

$$\begin{array}{llcccccc} \text{Mean} & \overline{d} & = & \overline{x} - \overline{y} & = & 10.62 - 21.71 & = & -11.09 \\ \text{Variance} & s_d^2 & = & s_x^2 + s_y^2 - 2s_{xy} & = & 2.3300^2 + 2.0684^2 - 2(-2.2447) & = & 14.1966 \\ \text{Std Dev} & s_d & := & \sqrt{s_d^2} & = & \sqrt{14.1966} & \approx & 3.7678 \end{array}$$

2[nd], compute paired t-test statistic value:

$$t = \frac{\overline{d} - \delta_0}{s_d/\sqrt{n}} = \frac{-11.09 - 0}{3.7678/\sqrt{10}} \approx \boxed{\mathbf{-9.3077}}$$

(d) Identify the two appropriate paired t-cutoffs and lookup/compute their values.

$$t^*_{n-1;\alpha/2} = t^*_{9;0.025} \overset{LOOKUP}{\approx} \boxed{\mathbf{2.262}} \qquad t^*_{n-1;1-\alpha/2} \overset{SYM}{=} -t^*_{n-1;\alpha/2} = -t^*_{9;0.025} \approx \boxed{\mathbf{-2.262}}$$

(e) Compute the appropriate P-value using software.

$$\text{P-value} = 2 \cdot [1 - \Phi_t(|t|; \nu = n-1)] = 2 \cdot [1 - \Phi_t(9.3077; \nu = 9)] \overset{SW}{\approx} \boxed{\mathbf{6.4810 \times 10^{-6}}}$$

(f) Using the computed paired t-cutoffs, render the appropriate decision.

Since $t < 0$ and $t \leq t^*_{n-1;1-\alpha/2}$, **Reject H_0 in favor of H_A**

(g) Using the computed P-value, render the appropriate decision.

Since P-value $\approx 6.4810 \times 10^{-6} < 0.05 = \alpha$, **Reject** H_0 in favor of H_A

The sample evidence is compelling enough to conclude that it's plausible that the average shear bond strengths of resin bonds differ by configuration.

(h) Construct the 95% paired t-CI for μ_D.

$$\overline{d} \pm t^*_{n-1;\alpha/2} \cdot \frac{s_d}{\sqrt{n}} \implies -11.09 \pm t^*_{9;0.025} \cdot \frac{3.7678}{\sqrt{10}} \overset{LOOKUP}{\implies} -11.09 \pm \mathbf{2.262} \cdot \frac{3.7678}{\sqrt{10}} \implies -11.09 \pm 2.6951$$

$\therefore$ 95% paired t-CI for μ_D is $\boxed{(\mathbf{-13.7851}, \mathbf{-8.3949})}$

Notice that the CI does not contain zero, which is expected since H_0 was rejected.