BASIC EXPERIMENTAL DESIGN TERMINOLOGY [DEVORE 9.3]

- **EXPERIMENTAL DESIGN:**
  - In most aspects of life, new products/services/techniques are developed as well as current ones being regularly refined and enhanced.
  - To determine if a new product, service or technique is more effective than one that is considered de facto standard or "status quo", an **experiment** must be performed in order to ascertain, based on performance measurements and the relevent subsequent statistical analyses, whether the new thing is demonstrably better than the current one.
  - Of course, the experiment must be carefully designed to ensure that any measured advantage of the new thing is not due to chance, bias or unexplained factors.

- **BASIC EXPERIMENTAL DESIGN TERMINOLOGY FOR 2-SAMPLE INFERENCE:**
  - The collection of 2 samples to determine cause & effect is an **experiment**.
  - Each data point of a sample is called an **observation** or **measurement**.
  - The dependent variable to be measured is called the **response**.
  - The person/object upon which the response is measured is a **subject**.
  - The manner of sample collection & grouping is called **experimental design**.
  - The main characteristic distinguishing the two samples is called the **factor**.
  - The factor's two particular values or settings are called its two **levels**.
  - Each sample corresponding to a level is called a **cell** or **treatment**.
  - **Nuisance factors** are uninteresting factors that influence the response.

FACTOR:	**CELL SIZE:**	**CELLS/TREATMENTS:**
Level 1	n	$x: \; x_1, \, x_2, \, \cdots, \, x_n$
Level 2	n	$y: \; y_1, \, y_2, \, \cdots, \, y_n$

FACTOR:	**CELL SIZE:**	**CELL/TREAT. MEAN:**	**CELL/TREAT. STD DEV:**
Level 1 (x)	n	$\overline{x}$	s_1
Level 2 (y)	n	$\overline{y}$	s_2

  - NOTE: **Observational studies** are not experiments as they don't allow cause-and-effect conclusions to be drawn.

PAIRED t-TESTS & PAIRED t-CI's [DEVORE 9.3]

- **PAIRED t-TEST FOR μ_D (UNKNOWN σ_1, σ_2):**

Population:	Two Normal Populations with unknown σ_1, σ_2
Realized Samples:	$\mathbf{x} := (x_1, x_2, \cdots, x_n)$ with mean $\overline{x}$, std dev $s_1, n \geq 2$ $\mathbf{y} := (y_1, y_2, \cdots, y_n)$ with mean $\overline{y}$, std dev $s_2, n \geq 2$
Difference Population:	$D := X - Y \implies D \sim \text{Normal}(\mu_D, \sigma_D^2)$ where $\mu_D = \mu_1 - \mu_2$ & $\sigma_D^2 = \sigma_1^2 + \sigma_2^2 - 2 \cdot \mathbb{C}[X, Y]$
Realized Pairs:	$(x_1, y_1), (x_2, y_2), \cdots (x_n, y_n)$
Realized Differences:	$\mathbf{d} := (d_1, d_2, \cdots, d_n)$ where $d_k := x_k - y_k$; $\overline{d} = \overline{x} - \overline{y}$ $s_d^2 = \underbrace{\frac{1}{n-1} \sum_{k=1}^n \left(d_k - \overline{d}\right)^2}_{\text{Given samples}} = \underbrace{s_x^2 + s_y^2 - 2s_{xy}}_{\text{Given covariance}} = \underbrace{s_x^2 + s_y^2 - 2r_{xy}s_x s_y}_{\text{Given correlation}}$
Test Statistic Value: $W(\mathbf{d}; \delta_0)$	$t = \dfrac{\overline{d} - \delta_0}{s_d/\sqrt{n}}$

HYPOTHESIS TEST:	**REJECTION REGION AT LVL α:**	**P-VALUE DETERMINATION:**
$H_0 : \mu_D = \delta_0$ vs. $H_A : \mu_D > \delta_0$	$t \geq t^*_{n-1;\alpha}$	P-value $= 1 - \Phi_t(t; \nu = n-1)$
$H_0 : \mu_D = \delta_0$ vs. $H_A : \mu_D < \delta_0$	$t \leq t^*_{n-1;1-\alpha}$	P-value $= \Phi_t(t; \nu = n-1)$
$H_0 : \mu_D = \delta_0$ vs. $H_A : \mu_D \neq \delta_0$	$t \leq t^*_{n-1;1-\alpha/2}$ or $t \geq t^*_{n-1;\alpha/2}$	P-value $= 2 \cdot [1 - \Phi_t(\lvert t \rvert; \nu = n-1)]$

Decision Rule: If P-value $\leq \alpha$ then reject H_0 in favor of H_A
If P-value $> \alpha$ then accept H_0 (i.e. fail to reject H_0)

- **PAIRED t-CI FOR μ_D (UNKNOWN σ_1, σ_2):**

Given two normal populations with means μ_1 and μ_2.

Let $x : x_1, x_2, \cdots, x_n$ and $y : y_1, y_2, \cdots, y_n$ be samples taken from the 1^{st} & 2^{nd} populations, respectively.

Moreover, suppose the two samples are paired as follows: $(x_1, y_1), (x_2, y_2), \cdots, (x_n, y_n)$

where the pairwise differences ($d_k := x_k - y_k$) are: $d : d_1, d_2, \cdots, d_n$

Then the $100(1-\alpha)\%$ **paired t-CI for** $\mu_D := \mu_1 - \mu_2$ is

$$\left(\overline{d} - t^*_{n-1;\alpha/2} \cdot \frac{s_d}{\sqrt{n}},\ \overline{d} + t^*_{n-1;\alpha/2} \cdot \frac{s_d}{\sqrt{n}}\right) \quad \text{OR MORE COMPACTLY} \quad \overline{d} \pm t^*_{n-1;\alpha/2} \cdot \frac{s_d}{\sqrt{n}}$$

where mean $\overline{d} = \overline{x} - \overline{y}$ and variance $s_d^2 = \underbrace{\frac{1}{n-1} \sum_{k=1}^{n} \left(d_k - \overline{d}\right)^2}_{\text{Given samples}} = \underbrace{s_x^2 + s_y^2 - 2s_{xy}}_{\text{Given covariance}} = \underbrace{s_x^2 + s_y^2 - 2r_{xy}s_x s_y}_{\text{Given correlation}}$

- **WHEN TO CHOOSE DEPENDENT (PAIRED) SAMPLES VS. INDEPENDENT SAMPLES:**
  - If entire body of subjects in experiment is largely heterogeneous and the pairing scheme chosen is reasonable such that any extraneous factors are similar between the two members in each pair (which suggests each pair's members bear small variance but large correlation), then use paired t-test as ESE will be smaller.
    - A blatant dependent sample would be observing/measuring a subject pre-treatment and then post-treatment.
    - Naturally correlated pairs include twins, father and son, two cats from the same litter, etc...
  - If entire body of subjects in experiment is largely homogeneous, then pairing will not be very effective – the tiny gains from pairing will be negated by the loss of half the dof's. Thus, use independent t-test instead.
  - Making the appropriate choice given the particular situation will result in narrower CI's and more-powerful tests.

EX 9.3.1: The lifetimes of two light bulb brands were measured via two dependent samples and summarized into this table:

BULB BRAND	(BULB LIFETIMES in yrs)
Brand 1 (x)	9.22, 9.07, 8.95, 8.98, 9.54
Brand 2 (y)	8.92, 8.88, 9.10, 8.71, 8.85

Do these samples suggest that the average bulb lifetimes differ by brand? (Use significance level $\alpha = 0.01$)

(a) Define μ_1, μ_2 and state the appropriate null hypothesis H_0 & alternative hypothesis H_A in terms of μ_1, μ_2.

(b) Define r.v. D, sample $\mathbf{d}$ and parameter μ_D. Then state the appropriate hypotheses in terms of μ_D.

(c) Compute the paired t-test statistic value for the hypothesis test concerning μ_D.

(d) Identify the two appropriate paired t-cutoffs and lookup/compute their values.

(e) Compute the appropriate P-value using software.

(f) Using the computed paired t-cutoffs, render the appropriate decision.

(g) Using the computed P-value, render the appropriate decision.

(h) Construct the 99% paired t-CI for μ_D.

(i) Identify some possible nuisance factors for this experiment.

EX 9.3.2: Dentists use resin composites and ceramic fillings among others for cavities in teeth. The shear bond strengths of resin composite-ceramic bonds formed from two possible configurations (conventional & all-composite) were measured (in MPa) via two dependent samples and summarized in the following table:

GROUP:	**SAMPLE SIZE:**	**MEAN:**	**STD DEV:**	**COVARIANCE:**
Conventional (x)	10	$\overline{x} = 10.62$	$s_x = 2.3300$	$s_{xy} = -2.2447$
All-Composite (y)	10	$\overline{y} = 21.71$	$s_y = 2.0684$	

This simplified table and all the details regarding the experiment can be found in the following paper:

A. Della Bona, R. van Noort, "Shear vs. Tensile Bond Strength of Resin Composite Bonded to Ceramic", *Journal of Dental Research*, **74** (1995), 1591-1596.

Does this experiment suggest that the average shear bond strengths of resin bonds differ by configuration?

(Use significance level $\alpha = 0.05$)

(a) Define μ_1, μ_2 and state the appropriate null hypothesis H_0 & alternative hypothesis H_A in terms of μ_1, μ_2.

(b) Define r.v. D, sample $\mathbf{d}$ and parameter μ_D. Then state the appropriate hypotheses in terms of μ_D.

(c) Compute the paired t-test statistic value for the hypothesis test concerning μ_D.

(d) Identify the two appropriate paired t-cutoffs and lookup/compute their values.

(e) Compute the appropriate P-value using software.

(f) Using the computed paired t-cutoffs, render the appropriate decision.

(g) Using the computed P-value, render the appropriate decision.

(h) Construct the 95% paired t-CI for μ_D.