EX 9.5.1: A manufacturing plant produces a certain type of brake pad used in big rig trucks.

A sample of 11 day-shift workers showed that the mean # of brake pads produced was 56 with a std dev of 4.2.

A sample of 16 night-shift workers showed that the mean # of brake pads produced was 48 with a std dev of 7.1.

Assume that the two samples are independent and the two corresponding populations are normally distributed.

Does the data suggest differing variability in the # brake pads produced by day-shift workers and night-shift workers?

(Use significance level $\alpha = 0.01$)

(a) Identify σ_+, σ_- and state the appropriate null hypothesis H_0 & alternative hypothesis H_A.

Identify & subscript the relevant sample variables, dof's and pop. std dev of smaller-std dev sample with minus signs.

Identify & subscript the relevant sample variables, dof's and pop. std dev of larger-std dev sample with plus signs.

$$s_+ = \max\{4.2, 7.1\} = 7.1 \implies n_+ = 16 \implies \nu_+ = n_+ - 1 = 15$$
$$s_- = \min\{4.2, 7.1\} = 4.2 \implies n_- = 11 \implies \nu_- = n_- - 1 = 10$$

$$\therefore \begin{array}{l} \sigma_+ \equiv \text{(Standard deviation of night-shift brake pad production)} \\ \sigma_- \equiv \text{(Standard deviation of day-shift brake pad production)} \end{array} \implies \boxed{\begin{array}{l} H_0 : \sigma_+ = \sigma_- \\ H_A : \sigma_+ \neq \sigma_- \end{array}}$$

(b) Compute the two F-test statistic values, f_+ & f_-, for this hypothesis test.

$$f_- = \frac{s_-^2}{s_+^2} = \frac{4.2^2}{7.1^2} \approx \boxed{\mathbf{0.34993}} \qquad f_+ = \frac{s_+^2}{s_-^2} = \frac{7.1^2}{4.2^2} \approx \boxed{\mathbf{2.85771}}$$

(c) Identify the two appropriate F-cutoffs, $f^*_{\nu_+,\nu_-;1-\alpha/2}$ & $f^*_{\nu_+,\nu_-;\alpha/2}$, and lookup their table values.

$$f^*_{\nu_+,\nu_-;1-\alpha/2} = \frac{1}{f^*_{\nu_-,\nu_+;\alpha/2}} = \frac{1}{f^*_{10,15;0.005}} \overset{LOOKUP}{\approx} \frac{1}{4.424} \approx \boxed{\mathbf{0.226}} \qquad f^*_{\nu_+,\nu_-;\alpha/2} = f^*_{15,10;0.005} \overset{LOOKUP}{\approx} \boxed{\mathbf{5.471}}$$

(d) Use software to compute the appropriate P-value.

$$\begin{aligned} \text{P-value} &= \Phi_F(f_-; \nu_-, \nu_+) + [1 - \Phi_F(f_+; \nu_+, \nu_-)] \\ &= \Phi_F(0.34993; 10, 15) + [1 - \Phi_F(2.85771; 15, 10)] \\ &\overset{SW}{\approx} 0.04931 + [1 - 0.95069] = \boxed{\mathbf{0.09862}} \end{aligned}$$

(e) Using the computed F-cutoffs, render the appropriate decision.

Observe that $f_- \approx 0.34993 > 0.226 \approx f^*_{\nu_+,\nu_-;1-\alpha/2} \implies f_- > f^*_{\nu_+,\nu_-;1-\alpha/2} \implies f_- \not\leq f^*_{\nu_+,\nu_-;1-\alpha/2}$

Observe that $f_+ \approx 2.85771 < 5.471 \approx f^*_{\nu_+,\nu_-;\alpha/2} \implies f_+ < f^*_{\nu_+,\nu_-;\alpha/2} \implies f_+ \not\geq f^*_{\nu_+,\nu_-;\alpha/2}$

$\therefore$ Since $f_- \not\leq f^*_{\nu_+,\nu_-;1-\alpha/2}$ and $f_+ \not\geq f^*_{\nu_+,\nu_-;\alpha/2}$, $\boxed{\textbf{Accept } H_0}$

There is not enough compelling evidence from the data to conclude that the variability in brake pad production differs between the day shift and night shift.

(f) Using the computed P-value, render the appropriate decision.

Since P-value $\approx 0.09862 > 0.01 = \alpha$, $\boxed{\textbf{Accept } H_0}$

(g) Construct the 99% F-CI for σ_+^2/σ_-^2.

$$\left(\frac{f_-}{f^*_{\nu_-,\nu_+;\alpha/2}}, \frac{f^*_{\nu_+,\nu_-;\alpha/2}}{f_+} \right) = \left(\frac{s_-^2/s_+^2}{f^*_{\nu_-,\nu_+;\alpha/2}}, \frac{f^*_{\nu_+,\nu_-;\alpha/2}}{s_+^2/s_-^2} \right) = \left(\frac{4.2^2/7.1^2}{4.424}, \frac{5.471}{7.1^2/4.2^2} \right) = \boxed{(\mathbf{0.0791},\ \mathbf{1.9145})}$$

Sanity Check: Notice that the F-CI contains the number one, which is expected since H_0 was accepted.

(h) Construct the 99% F-CI for σ_+/σ_-.

$$\left(\sqrt{\frac{f_-}{f^*_{\nu_-,\nu_+;\alpha/2}}}, \sqrt{\frac{f^*_{\nu_+,\nu_-;\alpha/2}}{f_+}} \right) = \left(\sqrt{\frac{s_-^2/s_+^2}{f^*_{\nu_-,\nu_+;\alpha/2}}}, \sqrt{\frac{f^*_{\nu_+,\nu_-;\alpha/2}}{s_+^2/s_-^2}} \right) = \left(\sqrt{\frac{4.2^2/7.1^2}{4.424}}, \sqrt{\frac{5.471}{7.1^2/4.2^2}} \right) = \boxed{(\mathbf{0.2812},\ \mathbf{1.3837})}$$

Sanity Check: Notice that the F-CI contains the number one, which is expected since H_0 was accepted.

EX 9.5.2: A manufacturing plant produces a certain type of brake pad used in big rig trucks.

A sample of 21 day-shift workers showed that the mean # of brake pads produced was 58 with a variance of 95.45.

A sample of 31 night-shift workers showed that the mean # of brake pads produced was 63 with a variance of 26.58.

Assume that the two samples are independent and the two corresponding populations are normally distributed.

Does the data suggest differing variability in the # brake pads produced by day-shift workers and night-shift workers?

(Use significance level $\alpha = 0.05$)

(a) Identify σ_+^2, σ_-^2 and state the appropriate null hypothesis H_0 & alternative hypothesis H_A.

Identify & subscript the relevant sample variables, dof's and pop. variance of larger-variance sample with plus signs.

Identify & subscript the relevant sample variables, dof's and pop. variance of smaller-variance sample with minus signs.

$$s_+^2 = \max\{95.45, 26.58\} = 95.45 \implies n_+ = 21 \implies \nu_+ = n_+ - 1 = 20$$
$$s_-^2 = \min\{95.45, 26.58\} = 26.58 \implies n_- = 31 \implies \nu_- = n_- - 1 = 30$$

$$\therefore \begin{array}{l} \sigma_+^2 \equiv (\text{Variance of day-shift brake pad production}) \\ \sigma_-^2 \equiv (\text{Variance of night-shift brake pad production}) \end{array} \implies \boxed{\begin{array}{l} H_0 : \sigma_+^2 = \sigma_-^2 \\ H_A : \sigma_+^2 \neq \sigma_-^2 \end{array}}$$

(b) Compute the two F-test statistic values, f_+ & f_-, for this hypothesis test.

$$f_- = \frac{s_-^2}{s_+^2} = \frac{26.58}{95.45} \approx \boxed{\mathbf{0.27847}} \qquad f_+ = \frac{s_+^2}{s_-^2} = \frac{95.45}{26.58} \approx \boxed{\mathbf{3.59105}}$$

(c) Identify the two appropriate F-cutoffs, $f^*_{\nu_+,\nu_-;1-\alpha/2}$ & $f^*_{\nu_+,\nu_-;\alpha/2}$, and lookup their table values.

$$f^*_{\nu_+,\nu_-;1-\alpha/2} = \frac{1}{f^*_{\nu_-,\nu_+;\alpha/2}} = \frac{1}{f^*_{30,20;0.025}} \overset{LOOKUP}{\approx} \frac{1}{2.349} \approx \boxed{\mathbf{0.426}} \qquad f^*_{\nu_+,\nu_-;\alpha/2} = f^*_{20,30;0.025} \overset{LOOKUP}{\approx} \boxed{\mathbf{2.195}}$$

(d) Use software to compute the appropriate P-value.

$$\begin{aligned} \text{P-value} &= \Phi_F(f_-;\nu_-,\nu_+) + [1 - \Phi_F(f_+;\nu_+,\nu_-)] \\ &= \Phi_F(0.27847; 30, 20) + [1 - \Phi_F(3.59105; 20, 30)] \\ &\overset{SW}{\approx} 0.00079787 + [1 - 0.9992021] = \boxed{\mathbf{0.001596}} \end{aligned}$$

(e) Using the computed F-cutoffs, render the appropriate decision.

Observe that $f_- \approx 0.27847 < 0.426 \approx f^*_{\nu_+,\nu_-;1-\alpha/2} \implies f_- \leq f^*_{\nu_+,\nu_-;1-\alpha/2}$

Observe that $f_+ \approx 3.59105 > 2.195 \approx f^*_{\nu_+,\nu_-;\alpha/2} \implies f_+ \geq f^*_{\nu_+,\nu_-;\alpha/2}$

$\therefore$ Since $f_- \leq f^*_{\nu_+,\nu_-;1-\alpha/2}$ and $f_+ \geq f^*_{\nu_+,\nu_-;\alpha/2}$, **Reject H_0 in favor of H_A**

The sample evidence is compelling enough to conclude that it's plausible that the variability in brake pad production differs between the day shift and night shift.

(f) Using the computed P-value, render the appropriate decision.

Since P-value $\approx 0.001596 < 0.05 = \alpha$, **Reject H_0 in favor of H_A**

(g) Construct the 95% F-CI for σ_+^2/σ_-^2.

$$\left(\frac{f_-}{f^*_{\nu_-,\nu_+;\alpha/2}}, \frac{f^*_{\nu_+,\nu_-;\alpha/2}}{f_+} \right) = \left(\frac{s_-^2/s_+^2}{f^*_{\nu_-,\nu_+;\alpha/2}}, \frac{f^*_{\nu_+,\nu_-;\alpha/2}}{s_+^2/s_-^2} \right) = \left(\frac{26.58/95.45}{2.349}, \frac{2.195}{95.45/26.58} \right) = \boxed{(\mathbf{0.1185},\ \mathbf{0.6112})}$$

Sanity Check: Notice that the F-CI does not contain the number one, which is expected since H_0 was rejected.

(h) Construct the 95% F-CI for σ_+/σ_-.

$$\left(\sqrt{\frac{f_-}{f^*_{\nu_-,\nu_+;\alpha/2}}}, \sqrt{\frac{f^*_{\nu_+,\nu_-;\alpha/2}}{f_+}} \right) = \left(\sqrt{\frac{s_-^2/s_+^2}{f^*_{\nu_-,\nu_+;\alpha/2}}}, \sqrt{\frac{f^*_{\nu_+,\nu_-;\alpha/2}}{s_+^2/s_-^2}} \right) = \left(\sqrt{\frac{26.58/95.45}{2.349}}, \sqrt{\frac{2.195}{95.45/26.58}} \right) = \boxed{(\mathbf{0.3443},\ \mathbf{0.7818})}$$

Sanity Check: Notice that the F-CI does not contain the number one, which is expected since H_0 was rejected.