# SNEDECOR'S $F$ DISTRIBUTION [DEVORE 9.5]

- **SNEDECOR'S $F$ DISTRIBUTION (DEFINITION):**

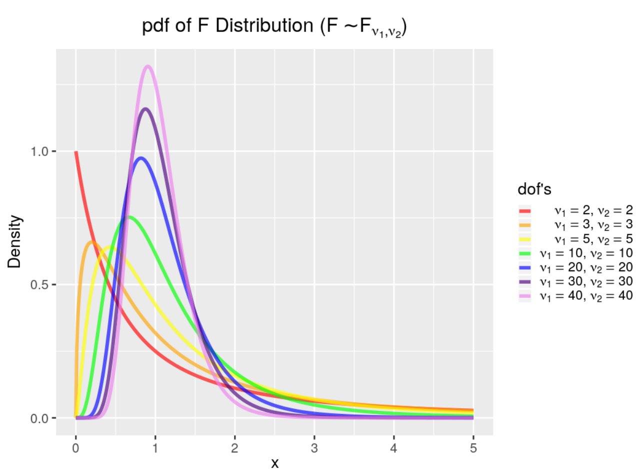
| | |
|---|---|
| Notation | $F \sim F_{\nu_1,\nu_2}$ |
| Parameters | $\nu_1 \equiv \#$ "Top" dof's $(\nu_1 = 1, 2, 3, \cdots)$<br>$\nu_2 \equiv \#$ "Bottom" dof's $(\nu_2 = 1, 2, 3, \cdots)$ |
| Support | $\text{Supp}(F) = (0, \infty)$ if $\nu_1 = 1$<br>$\text{Supp}(F) = [0, \infty)$ otherwise |
| pdf | $f_F(x; \nu_1, \nu_2) := \frac{(\nu_1/\nu_2)^{\nu_1/2}}{B(\nu_1/2, \nu_2/2)} \cdot x^{(\nu_1/2)-1} \left(1 + \frac{\nu_1}{\nu_2}x\right)^{-(\nu_1+\nu_2)/2}$ |
| cdf | $\Phi_F(x; \nu_1, \nu_2) = \frac{(\nu_1/\nu_2)^{\nu_1/2}}{B(\nu_1/2, \nu_2/2)} \int_0^x \xi^{(\nu_1/2)-1} \left(1 + \frac{\nu_1}{\nu_2}\xi\right)^{-(\nu_1+\nu_2)/2} d\xi$ |
| Mean | $\mathbb{E}[F] = \frac{\nu_2}{\nu_2 - 2}$, for $\nu_2 > 2$ |
| Variance | $\mathbb{V}[F] = \frac{2\nu_2^2(\nu_1+\nu_2-2)}{\nu_1(\nu_2-2)^2(\nu_2-4)}$, for $\nu_2 > 4$ |
| Model(s) | (Used exclusively for Statistical Inference) |

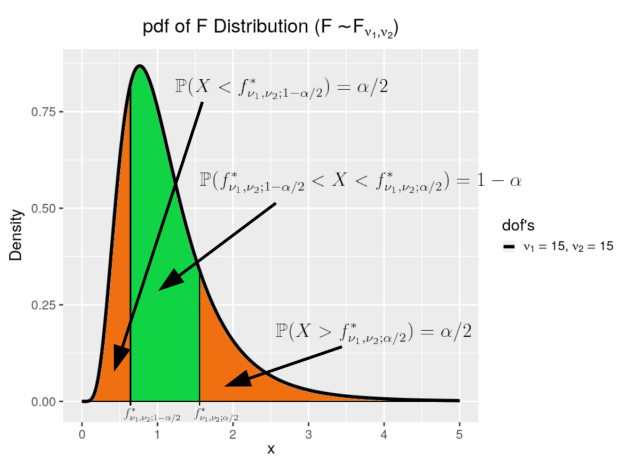
$B(\cdot,\cdot) \equiv$ Beta Function $\qquad \xi$ is the lowercase Greek letter "xi"

- **SNEDECOR'S $F$ DISTRIBUTION (PROPERTIES):**
  - The $F_{\nu_1,\nu_2}$ pdf curve is positively skewed.
  - Let random variable $X \sim F_{\nu_1,\nu_2}$. Then $\frac{1}{X} \sim F_{\nu_2,\nu_1}$
  - Let random variable $X \sim t_n$. Then $X^2 \sim F_{1,n}$
  - Let $X_1, X_2$ be independent rv's such that $X_1 \sim \chi^2_{\nu_1}$ & $X_2 \sim \chi^2_{\nu_2}$. Then $\frac{X_1/\nu_1}{X_2/\nu_2} \sim F_{\nu_1,\nu_2}$
  - Let $X_1, X_2$ be independent rv's s.t. $\begin{cases} X_1 \sim \text{Gamma}(\alpha_1, \beta_1) \\ X_2 \sim \text{Gamma}(\alpha_2, \beta_2) \end{cases}$. Then $\frac{\alpha_2\beta_1 X_1}{\alpha_1\beta_2 X_2} \sim F_{2\alpha_1,2\alpha_2}$

- **$F$-CUTOFFS (AKA $F$ CRITICAL VALUES):**
  - Value $f^*_{\nu_1,\nu_2;\alpha/2}$ is a **$F$-cutoff (AKA $F$ critical value)** of $F_{\nu_1,\nu_2}$ distribution such that $\mathbb{P}(F > f^*_{\nu_1,\nu_2;\alpha}) = \alpha$.
  - NOTE: Do not confuse $F$-cutoff $f^*_{\nu_1,\nu_2;\alpha}$ with $F$ percentile $f_{\nu_1,\nu_2;\alpha}$: $\quad \mathbb{P}(F \leq f_{\nu_1,\nu_2;\alpha}) = \alpha$
  - Lower-tail $F$-cutoffs can be found from upper-tail $F$-cutoffs: $\quad f^*_{\nu_1,\nu_2;1-\alpha} = \frac{1}{f^*_{\nu_2,\nu_1;\alpha}}$ (the order of the dof's switch!)

- **SNEDECOR'S $F$ DISTRIBUTION (EXAMPLE PDF PLOTS):**



# $F$-TESTS & $F$-CI'S FOR NORMAL POP. VARIANCE RATIOS [DEVORE 9.5]

- **COMPARING TWO POPULATION VARIANCES (MOTIVATION):**

  In most (if not all) situations, a higher variance is not desired as it indicates one or more of the following:

  - Less Reliability
  - Less Stability
  - Less Efficiency
  - Less Consistency
  - More Risk
  - More Uncertainty

  This is not surprising at all – we've encountered similar patterns before:

  - We prefer point estimators with smaller variances (e.g. UMVUE's.)
  - Smaller variances lead to narrower CI's at the same $\alpha$-level.
  - Smaller variances lead to more powerful hypothesis tests at the same significance level $\alpha$.

- **A STATISTIC RELATED TO THE $F$ DISTRIBUTION:**

  Let $\mathbf{X} := (X_1, \ldots, X_m)$ be a random sample from a Normal$(\mu_1, \sigma_1^2)$ population.

  Let $\mathbf{Y} := (Y_1, \ldots, Y_n)$ be a random sample from a Normal$(\mu_2, \sigma_2^2)$ population.

  Moreover, suppose random samples $\mathbf{X}$ & $\mathbf{Y}$ are independent of each other.

  Then the following statistic has an $F$ distribution: $\dfrac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} \sim F_{m-1,n-1}$

- **$F$-TEST FOR COMPARING NORMAL POPULATION VARIANCES:**

| Population: | Two Normal Populations with unknown $\sigma_+, \sigma_-$ |
|---|---|
| Realized Samples: | $\mathbf{x} := (x_1, x_2, \cdots, x_{n_+})$ with mean $\overline{x}$, std dev $s_+$<br>$\mathbf{y} := (y_1, y_2, \cdots, y_{n_-})$ with mean $\overline{y}$, std dev $s_-$<br>Samples $\mathbf{x}$ & $\mathbf{y}$ are independent of each other & $s_+ \geq s_-$ |
| Test Statistic Values:<br>$W_+(\mathbf{x}, \mathbf{y})$ & $W_-(\mathbf{x}, \mathbf{y})$ | $f_+ = \dfrac{s_+^2}{s_-^2}$, $f_- = \dfrac{s_-^2}{s_+^2}$, $\nu_+ = n_+ - 1$, $\nu_- = n_- - 1$ |

| HYPOTHESIS TEST: | REJECTION REGION AT LVL $\alpha$ : | P-VALUE DETERMINATION: |
|---|---|---|
| $H_0 : \sigma_+^2 = \sigma_-^2$ vs. $H_A : \sigma_+^2 > \sigma_-^2$ | $f_+ \geq f^*_{\nu_+,\nu_-;\alpha}$ | $1 - \Phi_F(f_+; \nu_+, \nu_-)$ |
| $H_0 : \sigma_+^2 = \sigma_-^2$ vs. $H_A : \sigma_+^2 \neq \sigma_-^2$ | $f_- \leq f^*_{\nu_+,\nu_-;1-\alpha/2}$ or $f_+ \geq f^*_{\nu_+,\nu_-;\alpha/2}$ | $\Phi_F(f_-; \nu_-, \nu_+) + [1 - \Phi_F(f_+; \nu_+, \nu_-)]$ |

- **$F$-CI's FOR $\sigma_+^2/\sigma_-^2$ AND $\sigma_+/\sigma_-$:**

  Given two normal populations with unknown variances $\sigma_+^2, \sigma_-^2$.

  Let $x_1, x_2, \cdots, x_{n_+}$ be a sample taken from $1^{st}$ population with variance $s_+^2$ and $\nu_+ = n_+ - 1$.

  Let $y_1, y_2, \cdots, y_{n_-}$ be a sample taken from $2^{nd}$ population with variance $s_-^2$ and $\nu_- = n_- - 1$.

  Moreover, the two samples are independent of each other with $s_+^2 \geq s_-^2$. Then:

  The $100(1-\alpha)\%$ **$F$-CI for $\sigma_+^2/\sigma_-^2$** is $\left( \dfrac{f_+}{f^*_{\nu_+,\nu_-;\alpha/2}}, \dfrac{f^*_{\nu_-,\nu_+;\alpha/2}}{f_-} \right) = \left( \dfrac{s_+^2/s_-^2}{f^*_{\nu_+,\nu_-;\alpha/2}}, \dfrac{f^*_{\nu_-,\nu_+;\alpha/2}}{s_-^2/s_+^2} \right)$

  The $100(1-\alpha)\%$ **$F$-CI for $\sigma_+/\sigma_-$** is $\left( \sqrt{\dfrac{f_+}{f^*_{\nu_+,\nu_-;\alpha/2}}}, \sqrt{\dfrac{f^*_{\nu_-,\nu_+;\alpha/2}}{f_-}} \right) = \left( \sqrt{\dfrac{s_+^2/s_-^2}{f^*_{\nu_+,\nu_-;\alpha/2}}}, \sqrt{\dfrac{f^*_{\nu_-,\nu_+;\alpha/2}}{s_-^2/s_+^2}} \right)$

## $(\alpha = \mathbf{0.1})$ SNEDECOR'S $F$-CUTOFFS, $f^*_{\nu_1,\nu_2;\alpha}$ $\mathbb{P}(F > f^*_{\nu_1,\nu_2;\alpha}) = \alpha$, $f^*_{\nu_1,\nu_2;1-\alpha} = \dfrac{1}{f^*_{\nu_2,\nu_1;\alpha}}$

| $\nu_2 \backslash \nu_1$ | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **12** | **15** | **18** | **20** | **30** | **40** | **60** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **2** | 9.000 | 9.162 | 9.243 | 9.293 | 9.326 | 9.349 | 9.367 | 9.381 | 9.392 | 9.408 | 9.425 | 9.436 | 9.441 | 9.458 | 9.466 | 9.475 |
| **3** | 5.462 | 5.391 | 5.343 | 5.309 | 5.285 | 5.266 | 5.252 | 5.240 | 5.230 | 5.216 | 5.200 | 5.190 | 5.184 | 5.168 | 5.160 | 5.151 |
| **4** | 4.325 | 4.191 | 4.107 | 4.051 | 4.010 | 3.979 | 3.955 | 3.936 | 3.920 | 3.896 | 3.870 | 3.853 | 3.844 | 3.817 | 3.804 | 3.790 |
| **5** | 3.780 | 3.619 | 3.520 | 3.453 | 3.405 | 3.368 | 3.339 | 3.316 | 3.297 | 3.268 | 3.238 | 3.217 | 3.207 | 3.174 | 3.157 | 3.140 |
| **6** | 3.463 | 3.289 | 3.181 | 3.108 | 3.055 | 3.014 | 2.983 | 2.958 | 2.937 | 2.905 | 2.871 | 2.848 | 2.836 | 2.800 | 2.781 | 2.762 |
| **7** | 3.257 | 3.074 | 2.961 | 2.883 | 2.827 | 2.785 | 2.752 | 2.725 | 2.703 | 2.668 | 2.632 | 2.607 | 2.595 | 2.555 | 2.535 | 2.514 |
| **8** | 3.113 | 2.924 | 2.806 | 2.726 | 2.668 | 2.624 | 2.589 | 2.561 | 2.538 | 2.502 | 2.464 | 2.438 | 2.425 | 2.383 | 2.361 | 2.339 |
| **9** | 3.006 | 2.813 | 2.693 | 2.611 | 2.551 | 2.505 | 2.469 | 2.440 | 2.416 | 2.379 | 2.340 | 2.312 | 2.298 | 2.255 | 2.232 | 2.208 |
| **10** | 2.924 | 2.728 | 2.605 | 2.522 | 2.461 | 2.414 | 2.377 | 2.347 | 2.323 | 2.284 | 2.244 | 2.215 | 2.201 | 2.155 | 2.132 | 2.107 |
| **12** | 2.807 | 2.606 | 2.480 | 2.394 | 2.331 | 2.283 | 2.245 | 2.214 | 2.188 | 2.147 | 2.105 | 2.075 | 2.060 | 2.011 | 1.986 | 1.960 |
| **15** | 2.695 | 2.490 | 2.361 | 2.273 | 2.208 | 2.158 | 2.119 | 2.086 | 2.059 | 2.017 | 1.972 | 1.941 | 1.924 | 1.873 | 1.845 | 1.817 |
| **18** | 2.624 | 2.416 | 2.286 | 2.196 | 2.130 | 2.079 | 2.038 | 2.005 | 1.977 | 1.933 | 1.887 | 1.854 | 1.837 | 1.783 | 1.754 | 1.723 |
| **20** | 2.589 | 2.380 | 2.249 | 2.158 | 2.091 | 2.040 | 1.999 | 1.965 | 1.937 | 1.892 | 1.845 | 1.811 | 1.794 | 1.738 | 1.708 | 1.677 |
| **30** | 2.489 | 2.276 | 2.142 | 2.049 | 1.980 | 1.927 | 1.884 | 1.849 | 1.819 | 1.773 | 1.722 | 1.686 | 1.667 | 1.606 | 1.573 | 1.538 |
| **40** | 2.440 | 2.226 | 2.091 | 1.997 | 1.927 | 1.873 | 1.829 | 1.793 | 1.763 | 1.715 | 1.662 | 1.625 | 1.605 | 1.541 | 1.506 | 1.467 |
| **60** | 2.393 | 2.177 | 2.041 | 1.946 | 1.875 | 1.819 | 1.775 | 1.738 | 1.707 | 1.657 | 1.603 | 1.564 | 1.543 | 1.476 | 1.437 | 1.395 |

## $(\alpha = \mathbf{0.05})$ SNEDECOR'S $F$-CUTOFFS, $f^*_{\nu_1,\nu_2;\alpha}$ $\mathbb{P}(F > f^*_{\nu_1,\nu_2;\alpha}) = \alpha$, $f^*_{\nu_1,\nu_2;1-\alpha} = \dfrac{1}{f^*_{\nu_2,\nu_1;\alpha}}$

| $\nu_2 \backslash \nu_1$ | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **12** | **15** | **18** | **20** | **30** | **40** | **60** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **2** | 19.00 | 19.16 | 19.25 | 19.30 | 19.33 | 19.35 | 19.37 | 19.38 | 19.40 | 19.41 | 19.43 | 19.44 | 19.45 | 19.46 | 19.47 | 19.48 |
| **3** | 9.552 | 9.277 | 9.117 | 9.013 | 8.941 | 8.887 | 8.845 | 8.812 | 8.786 | 8.745 | 8.703 | 8.675 | 8.660 | 8.617 | 8.594 | 8.572 |
| **4** | 6.944 | 6.591 | 6.388 | 6.256 | 6.163 | 6.094 | 6.041 | 5.999 | 5.964 | 5.912 | 5.858 | 5.821 | 5.803 | 5.746 | 5.717 | 5.688 |
| **5** | 5.786 | 5.409 | 5.192 | 5.050 | 4.950 | 4.876 | 4.818 | 4.772 | 4.735 | 4.678 | 4.619 | 4.579 | 4.558 | 4.496 | 4.464 | 4.431 |
| **6** | 5.143 | 4.757 | 4.534 | 4.387 | 4.284 | 4.207 | 4.147 | 4.099 | 4.060 | 4.000 | 3.938 | 3.896 | 3.874 | 3.808 | 3.774 | 3.740 |
| **7** | 4.737 | 4.347 | 4.120 | 3.972 | 3.866 | 3.787 | 3.726 | 3.677 | 3.637 | 3.575 | 3.511 | 3.467 | 3.445 | 3.376 | 3.340 | 3.304 |
| **8** | 4.459 | 4.066 | 3.838 | 3.687 | 3.581 | 3.500 | 3.438 | 3.388 | 3.347 | 3.284 | 3.218 | 3.173 | 3.150 | 3.079 | 3.043 | 3.005 |
| **9** | 4.256 | 3.863 | 3.633 | 3.482 | 3.374 | 3.293 | 3.230 | 3.179 | 3.137 | 3.073 | 3.006 | 2.960 | 2.936 | 2.864 | 2.826 | 2.787 |
| **10** | 4.103 | 3.708 | 3.478 | 3.326 | 3.217 | 3.135 | 3.072 | 3.020 | 2.978 | 2.913 | 2.845 | 2.798 | 2.774 | 2.700 | 2.661 | 2.621 |
| **12** | 3.885 | 3.490 | 3.259 | 3.106 | 2.996 | 2.913 | 2.849 | 2.796 | 2.753 | 2.687 | 2.617 | 2.568 | 2.544 | 2.466 | 2.426 | 2.384 |
| **15** | 3.682 | 3.287 | 3.056 | 2.901 | 2.790 | 2.707 | 2.641 | 2.588 | 2.544 | 2.475 | 2.403 | 2.353 | 2.328 | 2.247 | 2.204 | 2.160 |
| **18** | 3.555 | 3.160 | 2.928 | 2.773 | 2.661 | 2.577 | 2.510 | 2.456 | 2.412 | 2.342 | 2.269 | 2.217 | 2.191 | 2.107 | 2.063 | 2.017 |
| **20** | 3.493 | 3.098 | 2.866 | 2.711 | 2.599 | 2.514 | 2.447 | 2.393 | 2.348 | 2.278 | 2.203 | 2.151 | 2.124 | 2.039 | 1.994 | 1.946 |
| **30** | 3.316 | 2.922 | 2.690 | 2.534 | 2.421 | 2.334 | 2.266 | 2.211 | 2.165 | 2.092 | 2.015 | 1.960 | 1.932 | 1.841 | 1.792 | 1.740 |
| **40** | 3.232 | 2.839 | 2.606 | 2.449 | 2.336 | 2.249 | 2.180 | 2.124 | 2.077 | 2.003 | 1.924 | 1.868 | 1.839 | 1.744 | 1.693 | 1.637 |
| **60** | 3.150 | 2.758 | 2.525 | 2.368 | 2.254 | 2.167 | 2.097 | 2.040 | 1.993 | 1.917 | 1.836 | 1.778 | 1.748 | 1.649 | 1.594 | 1.534 |

## $(\alpha = \mathbf{0.05})$ SNEDECOR'S $F$-CUTOFFS, $f^*_{\nu_1,\nu_2;\alpha}$ $\quad \mathbb{P}(F > f^*_{\nu_1,\nu_2;\alpha}) = \alpha, \quad f^*_{\nu_1,\nu_2;1-\alpha} = \dfrac{1}{f^*_{\nu_2,\nu_1;\alpha}}$

| $\nu_2$ \ $\nu_1$ | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **12** | **15** | **18** | **20** | **30** | **40** | **60** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **2** | 19.00 | 19.16 | 19.25 | 19.30 | 19.33 | 19.35 | 19.37 | 19.38 | 19.40 | 19.41 | 19.43 | 19.44 | 19.45 | 19.46 | 19.47 | 19.48 |
| **3** | 9.552 | 9.277 | 9.117 | 9.013 | 8.941 | 8.887 | 8.845 | 8.812 | 8.786 | 8.745 | 8.703 | 8.675 | 8.660 | 8.617 | 8.594 | 8.572 |
| **4** | 6.944 | 6.591 | 6.388 | 6.256 | 6.163 | 6.094 | 6.041 | 5.999 | 5.964 | 5.912 | 5.858 | 5.821 | 5.803 | 5.746 | 5.717 | 5.688 |
| **5** | 5.786 | 5.409 | 5.192 | 5.050 | 4.950 | 4.876 | 4.818 | 4.772 | 4.735 | 4.678 | 4.619 | 4.579 | 4.558 | 4.496 | 4.464 | 4.431 |
| **6** | 5.143 | 4.757 | 4.534 | 4.387 | 4.284 | 4.207 | 4.147 | 4.099 | 4.060 | 4.000 | 3.938 | 3.896 | 3.874 | 3.808 | 3.774 | 3.740 |
| **7** | 4.737 | 4.347 | 4.120 | 3.972 | 3.866 | 3.787 | 3.726 | 3.677 | 3.637 | 3.575 | 3.511 | 3.467 | 3.445 | 3.376 | 3.340 | 3.304 |
| **8** | 4.459 | 4.066 | 3.838 | 3.687 | 3.581 | 3.500 | 3.438 | 3.388 | 3.347 | 3.284 | 3.218 | 3.173 | 3.150 | 3.079 | 3.043 | 3.005 |
| **9** | 4.256 | 3.863 | 3.633 | 3.482 | 3.374 | 3.293 | 3.230 | 3.179 | 3.137 | 3.073 | 3.006 | 2.960 | 2.936 | 2.864 | 2.826 | 2.787 |
| **10** | 4.103 | 3.708 | 3.478 | 3.326 | 3.217 | 3.135 | 3.072 | 3.020 | 2.978 | 2.913 | 2.845 | 2.798 | 2.774 | 2.700 | 2.661 | 2.621 |
| **12** | 3.885 | 3.490 | 3.259 | 3.106 | 2.996 | 2.913 | 2.849 | 2.796 | 2.753 | 2.687 | 2.617 | 2.568 | 2.544 | 2.466 | 2.426 | 2.384 |
| **15** | 3.682 | 3.287 | 3.056 | 2.901 | 2.790 | 2.707 | 2.641 | 2.588 | 2.544 | 2.475 | 2.403 | 2.353 | 2.328 | 2.247 | 2.204 | 2.160 |
| **18** | 3.555 | 3.160 | 2.928 | 2.773 | 2.661 | 2.577 | 2.510 | 2.456 | 2.412 | 2.342 | 2.269 | 2.217 | 2.191 | 2.107 | 2.063 | 2.017 |
| **20** | 3.493 | 3.098 | 2.866 | 2.711 | 2.599 | 2.514 | 2.447 | 2.393 | 2.348 | 2.278 | 2.203 | 2.151 | 2.124 | 2.039 | 1.994 | 1.946 |
| **30** | 3.316 | 2.922 | 2.690 | 2.534 | 2.421 | 2.334 | 2.266 | 2.211 | 2.165 | 2.092 | 2.015 | 1.960 | 1.932 | 1.841 | 1.792 | 1.740 |
| **40** | 3.232 | 2.839 | 2.606 | 2.449 | 2.336 | 2.249 | 2.180 | 2.124 | 2.077 | 2.003 | 1.924 | 1.868 | 1.839 | 1.744 | 1.693 | 1.637 |
| **60** | 3.150 | 2.758 | 2.525 | 2.368 | 2.254 | 2.167 | 2.097 | 2.040 | 1.993 | 1.917 | 1.836 | 1.778 | 1.748 | 1.649 | 1.594 | 1.534 |

## $(\alpha = \mathbf{0.025})$ SNEDECOR'S $F$-CUTOFFS, $f^*_{\nu_1,\nu_2;\alpha}$ $\quad \mathbb{P}(F > f^*_{\nu_1,\nu_2;\alpha}) = \alpha, \quad f^*_{\nu_1,\nu_2;1-\alpha} = \dfrac{1}{f^*_{\nu_2,\nu_1;\alpha}}$

| $\nu_2$ \ $\nu_1$ | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **12** | **15** | **18** | **20** | **30** | **40** | **60** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **2** | 39.00 | 39.17 | 39.25 | 39.30 | 39.33 | 39.36 | 39.37 | 39.39 | 39.40 | 39.41 | 39.43 | 39.44 | 39.45 | 39.46 | 39.47 | 39.48 |
| **3** | 16.04 | 15.44 | 15.10 | 14.88 | 14.73 | 14.62 | 14.54 | 14.47 | 14.42 | 14.34 | 14.25 | 14.20 | 14.17 | 14.08 | 14.04 | 13.99 |
| **4** | 10.65 | 9.979 | 9.605 | 9.364 | 9.197 | 9.074 | 8.980 | 8.905 | 8.844 | 8.751 | 8.657 | 8.592 | 8.560 | 8.461 | 8.411 | 8.360 |
| **5** | 8.434 | 7.764 | 7.388 | 7.146 | 6.978 | 6.853 | 6.757 | 6.681 | 6.619 | 6.525 | 6.428 | 6.362 | 6.329 | 6.227 | 6.175 | 6.123 |
| **6** | 7.260 | 6.599 | 6.227 | 5.988 | 5.820 | 5.695 | 5.600 | 5.523 | 5.461 | 5.366 | 5.269 | 5.202 | 5.168 | 5.065 | 5.012 | 4.959 |
| **7** | 6.542 | 5.890 | 5.523 | 5.285 | 5.119 | 4.995 | 4.899 | 4.823 | 4.761 | 4.666 | 4.568 | 4.501 | 4.467 | 4.362 | 4.309 | 4.254 |
| **8** | 6.059 | 5.416 | 5.053 | 4.817 | 4.652 | 4.529 | 4.433 | 4.357 | 4.295 | 4.200 | 4.101 | 4.034 | 3.999 | 3.894 | 3.840 | 3.784 |
| **9** | 5.715 | 5.078 | 4.718 | 4.484 | 4.320 | 4.197 | 4.102 | 4.026 | 3.964 | 3.868 | 3.769 | 3.701 | 3.667 | 3.560 | 3.505 | 3.449 |
| **10** | 5.456 | 4.826 | 4.468 | 4.236 | 4.072 | 3.950 | 3.855 | 3.779 | 3.717 | 3.621 | 3.522 | 3.453 | 3.419 | 3.311 | 3.255 | 3.198 |
| **12** | 5.096 | 4.474 | 4.121 | 3.891 | 3.728 | 3.607 | 3.512 | 3.436 | 3.374 | 3.277 | 3.177 | 3.108 | 3.073 | 2.963 | 2.906 | 2.848 |
| **15** | 4.765 | 4.153 | 3.804 | 3.576 | 3.415 | 3.293 | 3.199 | 3.123 | 3.060 | 2.963 | 2.862 | 2.792 | 2.756 | 2.644 | 2.585 | 2.524 |
| **18** | 4.560 | 3.954 | 3.608 | 3.382 | 3.221 | 3.100 | 3.005 | 2.929 | 2.866 | 2.769 | 2.667 | 2.596 | 2.559 | 2.445 | 2.384 | 2.321 |
| **20** | 4.461 | 3.859 | 3.515 | 3.289 | 3.128 | 3.007 | 2.913 | 2.837 | 2.774 | 2.676 | 2.573 | 2.501 | 2.464 | 2.349 | 2.287 | 2.223 |
| **30** | 4.182 | 3.589 | 3.250 | 3.026 | 2.867 | 2.746 | 2.651 | 2.575 | 2.511 | 2.412 | 2.307 | 2.233 | 2.195 | 2.074 | 2.009 | 1.940 |
| **40** | 4.051 | 3.463 | 3.126 | 2.904 | 2.744 | 2.624 | 2.529 | 2.452 | 2.388 | 2.288 | 2.182 | 2.107 | 2.068 | 1.943 | 1.875 | 1.803 |
| **60** | 3.925 | 3.343 | 3.008 | 2.786 | 2.627 | 2.507 | 2.412 | 2.334 | 2.270 | 2.169 | 2.061 | 1.985 | 1.944 | 1.815 | 1.744 | 1.667 |

## ($\alpha = \mathbf{0.01}$) SNEDECOR'S $F$-CUTOFFS, $f^*_{\nu_1,\nu_2;\alpha}$ $\mathbb{P}(F > f^*_{\nu_1,\nu_2;\alpha}) = \alpha$, $f^*_{\nu_1,\nu_2;1-\alpha} = \dfrac{1}{f^*_{\nu_2,\nu_1;\alpha}}$

| $\nu_2$ \ $\nu_1$ | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **12** | **15** | **18** | **20** | **30** | **40** | **60** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **2** | 99.00 | 99.17 | 99.25 | 99.30 | 99.33 | 99.36 | 99.37 | 99.39 | 99.40 | 99.42 | 99.43 | 99.44 | 99.45 | 99.47 | 99.47 | 99.48 |
| **3** | 30.82 | 29.46 | 28.71 | 28.24 | 27.91 | 27.67 | 27.49 | 27.35 | 27.23 | 27.05 | 26.87 | 26.75 | 26.69 | 26.50 | 26.41 | 26.32 |
| **4** | 18.00 | 16.69 | 15.98 | 15.52 | 15.21 | 14.98 | 14.80 | 14.66 | 14.55 | 14.37 | 14.20 | 14.08 | 14.02 | 13.84 | 13.75 | 13.65 |
| **5** | 13.27 | 12.06 | 11.39 | 10.97 | 10.67 | 10.46 | 10.29 | 10.16 | 10.05 | 9.888 | 9.722 | 9.610 | 9.553 | 9.379 | 9.291 | 9.202 |
| **6** | 10.92 | 9.780 | 9.148 | 8.746 | 8.466 | 8.260 | 8.102 | 7.976 | 7.874 | 7.718 | 7.559 | 7.451 | 7.396 | 7.229 | 7.143 | 7.057 |
| **7** | 9.547 | 8.451 | 7.847 | 7.460 | 7.191 | 6.993 | 6.840 | 6.719 | 6.620 | 6.469 | 6.314 | 6.209 | 6.155 | 5.992 | 5.908 | 5.824 |
| **8** | 8.649 | 7.591 | 7.006 | 6.632 | 6.371 | 6.178 | 6.029 | 5.911 | 5.814 | 5.667 | 5.515 | 5.412 | 5.359 | 5.198 | 5.116 | 5.032 |
| **9** | 8.022 | 6.992 | 6.422 | 6.057 | 5.802 | 5.613 | 5.467 | 5.351 | 5.257 | 5.111 | 4.962 | 4.860 | 4.808 | 4.649 | 4.567 | 4.483 |
| **10** | 7.559 | 6.552 | 5.994 | 5.636 | 5.386 | 5.200 | 5.057 | 4.942 | 4.849 | 4.706 | 4.558 | 4.457 | 4.405 | 4.247 | 4.165 | 4.082 |
| **12** | 6.927 | 5.953 | 5.412 | 5.064 | 4.821 | 4.640 | 4.499 | 4.388 | 4.296 | 4.155 | 4.010 | 3.909 | 3.858 | 3.701 | 3.619 | 3.535 |
| **15** | 6.359 | 5.417 | 4.893 | 4.556 | 4.318 | 4.142 | 4.004 | 3.895 | 3.805 | 3.666 | 3.522 | 3.423 | 3.372 | 3.214 | 3.132 | 3.047 |
| **18** | 6.013 | 5.092 | 4.579 | 4.248 | 4.015 | 3.841 | 3.705 | 3.597 | 3.508 | 3.371 | 3.227 | 3.128 | 3.077 | 2.919 | 2.835 | 2.749 |
| **20** | 5.849 | 4.938 | 4.431 | 4.103 | 3.871 | 3.699 | 3.564 | 3.457 | 3.368 | 3.231 | 3.088 | 2.989 | 2.938 | 2.778 | 2.695 | 2.608 |
| **30** | 5.390 | 4.510 | 4.018 | 3.699 | 3.473 | 3.304 | 3.173 | 3.067 | 2.979 | 2.843 | 2.700 | 2.600 | 2.549 | 2.386 | 2.299 | 2.208 |
| **40** | 5.179 | 4.313 | 3.828 | 3.514 | 3.291 | 3.124 | 2.993 | 2.888 | 2.801 | 2.665 | 2.522 | 2.421 | 2.369 | 2.203 | 2.114 | 2.019 |
| **60** | 4.977 | 4.126 | 3.649 | 3.339 | 3.119 | 2.953 | 2.823 | 2.718 | 2.632 | 2.496 | 2.352 | 2.251 | 2.198 | 2.028 | 1.936 | 1.836 |

## ($\alpha = \mathbf{0.005}$) SNEDECOR'S $F$-CUTOFFS, $f^*_{\nu_1,\nu_2;\alpha}$ $\mathbb{P}(F > f^*_{\nu_1,\nu_2;\alpha}) = \alpha$, $f^*_{\nu_1,\nu_2;1-\alpha} = \dfrac{1}{f^*_{\nu_2,\nu_1;\alpha}}$

| $\nu_2$ \ $\nu_1$ | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **12** | **15** | **18** | **20** | **30** | **40** | **60** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **2** | 199.0 | 199.2 | 199.2 | 199.3 | 199.3 | 199.4 | 199.4 | 199.4 | 199.4 | 199.4 | 199.4 | 199.4 | 199.4 | 199.5 | 199.5 | 199.5 |
| **3** | 49.80 | 47.47 | 46.19 | 45.39 | 44.84 | 44.43 | 44.13 | 43.88 | 43.69 | 43.39 | 43.08 | 42.88 | 42.78 | 42.47 | 42.31 | 42.15 |
| **4** | 26.28 | 24.26 | 23.15 | 22.46 | 21.97 | 21.62 | 21.35 | 21.14 | 20.97 | 20.70 | 20.44 | 20.26 | 20.17 | 19.89 | 19.75 | 19.61 |
| **5** | 18.31 | 16.53 | 15.56 | 14.94 | 14.51 | 14.20 | 13.96 | 13.77 | 13.62 | 13.38 | 13.15 | 12.98 | 12.90 | 12.66 | 12.53 | 12.40 |
| **6** | 14.54 | 12.92 | 12.03 | 11.46 | 11.07 | 10.79 | 10.57 | 10.39 | 10.25 | 10.03 | 9.814 | 9.664 | 9.589 | 9.358 | 9.241 | 9.122 |
| **7** | 12.40 | 10.88 | 10.05 | 9.522 | 9.155 | 8.885 | 8.678 | 8.514 | 8.380 | 8.176 | 7.968 | 7.826 | 7.754 | 7.534 | 7.422 | 7.309 |
| **8** | 11.04 | 9.596 | 8.805 | 8.302 | 7.952 | 7.694 | 7.496 | 7.339 | 7.211 | 7.015 | 6.814 | 6.678 | 6.608 | 6.396 | 6.288 | 6.177 |
| **9** | 10.11 | 8.717 | 7.956 | 7.471 | 7.134 | 6.885 | 6.693 | 6.541 | 6.417 | 6.227 | 6.032 | 5.899 | 5.832 | 5.625 | 5.519 | 5.410 |
| **10** | 9.427 | 8.081 | 7.343 | 6.872 | 6.545 | 6.302 | 6.116 | 5.968 | 5.847 | 5.661 | 5.471 | 5.340 | 5.274 | 5.071 | 4.966 | 4.859 |
| **12** | 8.510 | 7.226 | 6.521 | 6.071 | 5.757 | 5.525 | 5.345 | 5.202 | 5.085 | 4.906 | 4.721 | 4.595 | 4.530 | 4.331 | 4.228 | 4.123 |
| **15** | 7.701 | 6.476 | 5.803 | 5.372 | 5.071 | 4.847 | 4.674 | 4.536 | 4.424 | 4.250 | 4.070 | 3.946 | 3.883 | 3.687 | 3.585 | 3.480 |
| **18** | 7.215 | 6.028 | 5.375 | 4.956 | 4.663 | 4.445 | 4.276 | 4.141 | 4.030 | 3.860 | 3.683 | 3.560 | 3.498 | 3.303 | 3.201 | 3.096 |
| **20** | 6.986 | 5.818 | 5.174 | 4.762 | 4.472 | 4.257 | 4.090 | 3.956 | 3.847 | 3.678 | 3.502 | 3.380 | 3.318 | 3.123 | 3.022 | 2.916 |
| **30** | 6.355 | 5.239 | 4.623 | 4.228 | 3.949 | 3.742 | 3.580 | 3.450 | 3.344 | 3.179 | 3.006 | 2.885 | 2.823 | 2.628 | 2.524 | 2.415 |
| **40** | 6.066 | 4.976 | 4.374 | 3.986 | 3.713 | 3.509 | 3.350 | 3.222 | 3.117 | 2.953 | 2.781 | 2.661 | 2.598 | 2.401 | 2.296 | 2.184 |
| **60** | 5.795 | 4.729 | 4.140 | 3.760 | 3.492 | 3.291 | 3.134 | 3.008 | 2.904 | 2.742 | 2.570 | 2.450 | 2.387 | 2.187 | 2.079 | 1.962 |

**EX 9.5.1:** A manufacturing plant produces a certain type of brake pad used in big rig trucks.

A sample of 11 day-shift workers showed that the mean # of brake pads produced was 56 with a std dev of 4.2.

A sample of 16 night-shift workers showed that the mean # of brake pads produced was 48 with a std dev of 7.1.

Assume that the two samples are independent and the two corresponding populations are normally distributed.

Does the data suggest differing variability in the # brake pads produced by day-shift workers and night-shift workers?

(Use significance level $\alpha = 0.01$)

(a) Identify $\sigma_+, \sigma_-$ and state the appropriate null hypothesis $H_0$ & alternative hypothesis $H_A$.

(b) Compute the two $F$-test statistic values, $f_+$ & $f_-$, for this hypothesis test.

(c) Identify the two appropriate $F$-cutoffs, $f^*_{\nu_+,\nu_-;1-\alpha/2}$ & $f^*_{\nu_+,\nu_-;\alpha/2}$, and lookup their table values.

(d) Use software to compute the appropriate P-value.

(e) Using the computed $F$-cutoffs, render the appropriate decision.

(f) Using the computed P-value, render the appropriate decision.

(g) Construct the 99% $F$-CI for $\sigma_+^2/\sigma_-^2$.

(h) Construct the 99% $F$-CI for $\sigma_+/\sigma_-$.

**EX 9.5.2:** A manufacturing plant produces a certain type of brake pad used in big rig trucks.

A sample of 21 day-shift workers showed that the mean # of brake pads produced was 58 with a variance of 95.45.

A sample of 31 night-shift workers showed that the mean # of brake pads produced was 63 with a variance of 26.58.

Assume that the two samples are independent and the two corresponding populations are normally distributed.

Does the data suggest differing variability in the # brake pads produced by day-shift workers and night-shift workers?

(Use significance level $\alpha = 0.05$)

(a) Identify $\sigma_+^2, \sigma_-^2$ and state the appropriate null hypothesis $H_0$ & alternative hypothesis $H_A$.

(b) Compute the two $F$-test statistic values, $f_+$ & $f_-$, for this hypothesis test.

(c) Identify the two appropriate $F$-cutoffs, $f^*_{\nu_+,\nu_-;1-\alpha/2}$ & $f^*_{\nu_+,\nu_-;\alpha/2}$, and lookup their table values.

(d) Use software to compute the appropriate P-value.

(e) Using the computed $F$-cutoffs, render the appropriate decision.

(f) Using the computed P-value, render the appropriate decision.

(g) Construct the 95% $F$-CI for $\sigma_+^2/\sigma_-^2$.

(h) Construct the 95% $F$-CI for $\sigma_+/\sigma_-$.