MATH 1300-201: EXAM 2 INFO/LOGISTICS/ADVICE

- **INFO:**

WHEN:	Friday (07/17) at 8:00am	DURATION:	110 mins
PROBLEM COUNT:	Six	BONUS COUNT:	Two

  - TOPICS CANDIDATE FOR THE EXAM: ("PIRNOT" means the textbook, 5^{th} ed.)
    - PIRNOT 10.1: Hamilton's Method, Alabama Paradox
    - PIRNOT 10.3: New States Paradox, Jefferson's Mtd, Adams' Mtd, Webster's Mtd
    - PIRNOT 11.1: Plurality Mtd, Borda Count Mtd, Pairwise Comparison Mtd
    - PIRNOT 11.2: Majority Criterion, Condorcet Criterion, Independence-of-Irrelevant-Alternatives (IIA) Crit.
    - PIRNOT 11.3: Weighted Voting Systems, Coalitions, Banzhaf Power Index
    - REMARK: **Do not Memorize Formulas – A Formula Sheet will be provided (next two pages)**
  - TOPICS CANDIDATE FOR BONUS QUESTIONS:
    - PIRNOT 11.1: Plurality-with-Elimination Method
    - ?????
    - REMARK: **Maximum Bonus Points Possible = 30**
  - TOPICS NOT COVERED AT ALL:
    - PIRNOT 10.1: Measuring Fairness & Average Constituency (EXAMPLES 2-4, pgs 521-522)
    - PIRNOT 10.2: Huntington-Hill Method (entire section)
    - PIRNOT 10.3: Population Paradox (EXAMPLE 3, pgs 537-538), Quota Rule
    - PIRNOT 10.4: Fair Division (entire section)
    - PIRNOT 11.2: Monotonicity Criterion (EXAMPLE 5, pgs 577-579)
    - PIRNOT 11.4: The Shapley-Shubik Index (entire section)

- **LOGISTICS:**
  - **All you need to bring are pencil(s), eraser(s), calculators(s) & your Raidercard**.
  - Clear your desk of everything except pencil(s), eraser(s), calculator(s).
  - **Backpacks are to placed at the front of the classroom**.
  - **Formula Sheet (next two pages) will be provided**.
  - **Books, notes, notecards NOT PERMITTED**.
  - Mobile devices (phones, tablets, PC's, music, headphones, ...) are to be shut off and put away.
  - Tissues will be furnished – for allergies, not for sobbing.
  - **When you turn in your exam, be prepared to show me your Raidercard if I don't recognize you**.
  - **If you ask to use the restroom during the exam, either hold it or turn in your exam for grading**.

- **ADVICE:**
  - **Use the restroom before the exam, if needed**.
  - **Do not be late – set your wake-up alarms (consider using your cellphone as a backup alarm)**.
  - Review past homework, and perhaps even work some similar problems in the textbook.
  - Review the slides.
  - **Know how the use all formulas on the provided Formula Sheet (next two pages)**
  - **If you need more review, show up to extended office hours on Thursday (07/16) in MATH 003A**.
  - **SHOW APPROPRIATE WORK! Attempt bonus questions**.

MATH 1300: EXAM 2 FORMULA SHEET

NOTATION FOR ROUNDING:

Always Round Down: $\lfloor 3 \rfloor = 3 \quad \lfloor 3.1 \rfloor = 3 \quad \lfloor 3.5 \rfloor = 3 \quad \lfloor 3.9 \rfloor = 3$

Always Round Up: $\lceil 3 \rceil = 3 \quad \lceil 3.1 \rceil = 4 \quad \lceil 3.5 \rceil = 4 \quad \lceil 3.9 \rceil = 4$

Round to Nearest Integer: $[\![3]\!] = 3 \quad [\![3.1]\!] = 3 \quad [\![3.5]\!] = 4 \quad [\![3.9]\!] = 4$

PIRNOT 10.1:

$N \equiv$ # of states, $M \equiv$ # of seats, $P_k \equiv$ Population of the k^{th} state, $P \equiv$ Total Population, $D \equiv$ Standard Divisor

$Q_k \equiv$ Quota of the k^{th} state, $A_k \equiv$ Apportionment of the k^{th} state

- Hamilton's Method:

STEP 1:	Compute **standard divisor**	$D = \dfrac{P}{M}$
STEP 2:	Compute quotas, **always rounding down**	$Q_k = \left\lfloor \dfrac{P_k}{D} \right\rfloor$
STEP 3:	Compute the **total quota**	$T = \sum_{k=1}^{N} Q_k$
STEP 4:	If $T < M$, assign each of the $(M - T)$ surplus seats (one at a time) to the states having **quotas** with the **largest fractional parts**	$A_k = Q_k + (surplus)$

- Alabama Paradox: Increasing the total seats may **decrease** a state's apportionment.
- New States Paradox: The addition of a new state with its fair share of seats can affect apportionment of other states.

PIRNOT 10.3:

- Jefferson's Method:

STEP 1:	Compute **standard divisor**	$D = \dfrac{P}{M}$
STEP 2:	Compute divisor	$D^* = D\left[1 + \alpha\left(\dfrac{N}{M}\right)\right]$
STEP 3:	Compute quotas, **rounding down**	$Q_k = \left\lfloor \dfrac{P_k}{D^*} \right\rfloor$
STEP 4:	The apportionment is precisely the quota	$A_k = Q_k$

- Adams' Method:

STEP 1:	Compute **standard divisor**	$D = \dfrac{P}{M}$
STEP 2:	Compute divisor	$D^* = D\left[1 + \alpha\left(\dfrac{N}{M}\right)\right]$
STEP 3:	Compute quotas, **rounding up**	$Q_k = \left\lceil \dfrac{P_k}{D^*} \right\rceil$
STEP 4:	The apportionment is precisely the quota	$A_k = Q_k$

- Webster's Method:

STEP 1:	Compute **standard divisor**	$D = \dfrac{P}{M}$
STEP 2:	Compute divisor	$D^* = D\left[1 + \alpha\left(\dfrac{N}{M}\right)\right]$
STEP 3:	Compute quotas, **rounding as usual**	$Q_k = \left[\!\left[\dfrac{P_k}{D^*} \right]\!\right]$
STEP 4:	The apportionment is precisely the quota	$A_k = Q_k$

PIRNOT 11.1:

- Plurality Method:

  SETUP: Single-Winner Election has k candidates
  PROCESS: Each voter votes for one candidate
  WINNER: Candidate receiving the **most votes**

- Borda Count Method:

  SETUP: Single-Winner Election has k candidates
  PROCESS: (1) Each voter ranks all candidates as follows:
  The 1^{st} choice receives k points
  The 2^{nd} choice receives $(k-1)$ points
  The 3^{rd} choice receives $(k-2)$ points
  $\vdots$
  The last choice receives 1 point
  (2) For each candidate, compute the total sum of points
  WINNER: Candidate receiving the **most total points**

- Plurality-with-Elimination Method:

  SETUP: Single-Winner Election has k candidates
  PROCESS: (0) Compute total votes & # votes needed for a majority $= \left\lceil \frac{\text{(total votes)}}{2} \right\rceil$
  (1) If no candidate receives a majority of votes, then drop candidate(s) with fewest votes from the ballot
  (2) Conduct a new election round with updated ballot
  Assume voters don't change their preferences each round
  (3) Repeat (1)-(2) until a candidate receives a majority
  WINNER: Candidate receiving a **majority of votes**

- Pairwise Comparison Method:

  SETUP: Single-Winner Election has k candidates
  PROCESS: (1) Voters rank all candidates
  (2) Pit candidates A and B "head-to-head"
  Count how many voters prefer A to B
  Count how many voters prefer B to A
  If A and B are tied, then each receives 1/2 point
  Else the more preferred candidate receives 1 point
  and the less preferred candidate receives 0 points
  (3) Repeat Step (2) for each pair of candidates
  WINNER: Candidate receiving the **most points**

PIRNOT 11.2:

- Majority Criterion:

  If a majority of the voters rank a candidate as their 1^{st} choice, then that candidate should win the election.

- Condorcet Criterion:

  If candidate X can defeat each of the other candidates head-to-head, then candidate X is the winner of the election.

- IIA Criterion:

  If candidate X wins, some nonwinner(s) are removed from ballot, and a recount is done, then candidate X still wins.

PIRNOT 11.3:

- A **weighted voting system** with N voters is described by the following:

  $$[(\text{quota}) : (\text{weight of voter 1}), (\text{weight of voter 2}), \ldots, (\text{weight of voter } N)] \quad \equiv \quad [Q : w_1, w_2, \ldots, w_N]$$

  The **quota** Q is the # of votes needed in this system to get a **motion** or **resolution** (i.e. vote "Yes" or "No") passed.

  The **weights** $w_1, w_2, \ldots, w_N$ are the amount of votes controlled by voter 1, voter 2, ..., voter N

- A **coalition** is a group of voters who vote the same way.

  A **coalition's weight** W is the sum of the weights of all voters in the coalition: $W = \sum_{k=1}^{N} w_k$

  A coalition is called a **winning coalition** if the coalition's weight is greater than or equal to the quota: $W \geq Q$

  Voter k in a coalition is a **dictator** if voter k has total control: $w_k \geq Q$

  Voter k in a winning coalition is **critical** if the coalition needs voter k to win: $W - w_k < Q$

- (Banzhaf Power for Voter k) $= \dfrac{\text{\# times voter } k \text{ is } \textbf{critical} \text{ in winning coalitions}}{\text{Total \# times voters are } \textbf{critical} \text{ in winning coalitions}}$: $B_k = \dfrac{C_k}{T}$, where $T = \sum_{k=1}^{N} C_k$