

# MATH 1300-201: FINAL EXAM INFO/LOGISTICS/ADVICE

## • INFO:

WHEN:	Thursday (08/06) at 11:00am	DURATION:	150 mins
PROBLEM COUNT:	Eleven	BONUS COUNT:	Two

There will be three Ch13 problems, three Ch14 problems, and one problem from each of the past chapters.

- TOPICS CANDIDATE FOR THE EXAM: (“PIRNOT” means the textbook, 5<sup>th</sup> ed.)
  - \* (All ’TOPICS CANDIDATE FOR THE EXAM’s from Info/Logistics/Advice for EXAMS 1,2,3)
  - \* PIRNOT 13.1: Probability, Sample Spaces, Outcomes, Events, Odds
  - \* PIRNOT 13.2: Complements of Events, Unions of Events, Intersections of Events
  - \* PIRNOT 13.3: Conditional Probability, Probability Trees, Independent Events
  - \* PIRNOT 13.4: Expected Value, Fairness of Games of Chance
  - \* PIRNOT 14.1: Frequency Tables, Histograms, Stem-and-Leaf Displays, Back-to-Back Stem-and-Leaves
  - \* PIRNOT 14.2: Mean, Median, Mode
  - \* PIRNOT 14.3: Range, Standard Deviation
  - \* PIRNOT 14.4: Normal Distributions, 68-95-99.7 Rule
  - \* REMARK: Do not Memorize Formulas – A Formula Sheet will be provided (next two pages)
- TOPICS CANDIDATE FOR BONUS QUESTIONS:
  - \* ?????
  - \* ?????
  - \* REMARK: Maximum Bonus Points Possible = 20
- TOPICS NOT COVERED AT ALL:
  - \* (All ’TOPICS NOT COVERED AT ALL’ sections from Info/Logistics/Advice for EXAMS 1,2,3)
  - \* CHAPTER 13: Probability Problems requiring ”Advanced Counting” (i.e. **Combinations & Permutations**)
  - \* PIRNOT 13.1: Genetics (EXAMPLE 7, pg 652)
  - \* PIRNOT 13.4: Expected Value in Business (EXAMPLE 6, pg 687)
  - \* PIRNOT 13.5: Binomial Experiments (entire section)
  - \* PIRNOT 14.2: 5-Number Summaries (EXAMPLE 6, pg 720), Box-and-Whisker Plots (FIGURE 14.9, pg 721)
  - \* PIRNOT 14.3: Coefficient of Variation (EXAMPLE 4, pgs 733-734)
  - \* PIRNOT 14.4: z-Scores (pgs 741-747)
  - \* PIRNOT 14.5: Linear Correlation (entire section)

## • LOGISTICS:

- All you need to bring are pencil(s), eraser(s), calculators(s) & your Raidercard.
- Clear your desk of everything except pencil(s), eraser(s), calculator(s).
- Backpacks are to placed at the front of the classroom.
- Formula Sheet will be provided (see next 7 pages).
- Books, notes, notecards NOT PERMITTED.
- Mobile devices (phones, tablets, PC’s, music, headphones, ...) are to be shut off and put away.
- Tissues will be furnished – for allergies, not for sobbing.
- When you turn in your exam, be prepared to show me your Raidercard if I don’t recognize you.
- If you ask to use the restroom during the exam, either hold it or turn in your exam for grading.

## • ADVICE:

- (See the ’ADVICE’ section of the Info/Logistics/Advice for EXAMS 1,2,3)

# MATH 1300: FINAL EXAM FORMULA SHEET

## PIRNOT 8.1:

- $(\text{Percent Change}) = \frac{(\text{New Amount}) - (\text{Base Amount})}{(\text{Base Amount})}$
  - $(\text{New Amount}) = (\text{Base Amount}) \times [1 + (\text{Percent Change})]$
  - $(\text{Base Amount}) = \frac{(\text{New Amount})}{1 + (\text{Percent Change})}$
- 

## PIRNOT 8.2:

$I \equiv$  Amount of Simple Interest,       $FV \equiv$  Future Value,  
 $P \equiv$  Principal (Amount Borrowed),       $r \equiv$  Annual Interest Rate,  
 $m \equiv$  Frequency of Compounding per Year,       $t \equiv$  Time (in **years**)

- Simple Interest:  $I = Prt$
  - Simple Interest:  $FV = P(1 + rt)$
  - Compound Interest:  $FV = P \left(1 + \frac{r}{m}\right)^n$ , where  $n = mt$
- 

## PIRNOT 8.3:

- Add-On Interest Method:  $(\text{Monthly Payment}) = \frac{P + I}{n}$ , where  $I = Prt$ ,       $n \equiv$  # Payments
- Unpaid Balance Method:  $(\text{Finance Charge for Next Month}) = (\text{Unpaid Balance}) \times r \times (1 \text{ Month})$

$$(\text{Unpaid Balance}) = \left( \begin{array}{c} \text{Last} \\ \text{Month's} \\ \text{Balance} \end{array} \right) + \left( \begin{array}{c} \text{Finance Charge} \\ \text{on Last Month's} \\ \text{Balance} \end{array} \right) + (\text{Purchases}) - (\text{Returns}) - (\text{Payments})$$

$$(\text{Finance Charge on Last Month's Balance}) = (\text{Last Month's Balance}) \times r \times (1 \text{ Month})$$

- Average Daily Balance Method:  $(\text{Finance Charge for Next Month}) = (\text{Avg Daily Balance}) \times r \times (\# \text{ Days in Month})$

$$(\text{Avg Daily Balance}) = \frac{(\text{Total Daily Balance from Table})}{(\# \text{ Days in Month})}$$

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## PIRNOT 8.4:

$m \equiv$  # of Payments per Year,       $t \equiv$  Time (in **years**)

- Annuity:  $FV = \frac{mR}{r} \left[ \left(1 + \frac{r}{m}\right)^n - 1 \right]$ , where  $R \equiv$  Payment into the Annuity each Compounding Period,       $n = mt$
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## PIRNOT 8.5:

- Amortized Loan:  $P \left(1 + \frac{r}{m}\right)^n = \frac{mR}{r} \left[ \left(1 + \frac{r}{m}\right)^n - 1 \right]$
- Amortization Schedule:  $(\text{Monthly Interest Rate}) = \frac{1}{12} \times (\text{Annual Interest Rate})$

For each month,

$$(\text{Interest Paid}) = (\text{Last Balance}) \times (\text{Monthly Interest Rate}) \times (1 \text{ Month})$$

$$(\text{Monthly Payment}) - (\text{Interest Paid}) = (\text{Paid on Principal})$$

$$(\text{Remaining Balance}) = (\text{Last Balance}) - (\text{Paid on Principal})$$

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## PIRNOT 8.6:

$FC \equiv$  Finance Charge,       $FCPH \equiv$  Finance Charge per \$100 Financed,       $n \equiv$  # Payments

- Computing APR using a Table (which will be provided):

STEP 1: Compute  $\mathbf{FCPH} = \frac{(\text{Finance Charge})}{(\text{Amount Borrowed})} \times 100 = \frac{FC}{P} \times 100$

STEP 2: Find the **closest entry** in "n Payments" Row of provided table to **FCPH**

STEP 3: The **column heading of the table entry** is the APR

- Estimating APR using the Formula:  $\text{APR} \approx \frac{2nr}{n+1}$
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## NOTATION FOR ROUNDING:

Always Round Down:  $\lfloor 3 \rfloor = 3$   $\lfloor 3.1 \rfloor = 3$   $\lfloor 3.5 \rfloor = 3$   $\lfloor 3.9 \rfloor = 3$

Always Round Up:  $\lceil 3 \rceil = 3$   $\lceil 3.1 \rceil = 4$   $\lceil 3.5 \rceil = 4$   $\lceil 3.9 \rceil = 4$

Round to Nearest Integer:  $\llbracket 3 \rrbracket = 3$   $\llbracket 3.1 \rrbracket = 3$   $\llbracket 3.5 \rrbracket = 4$   $\llbracket 3.9 \rrbracket = 4$

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### PIRNOT 10.1:

$N \equiv$  # of states,  $M \equiv$  # of seats,  $P_k \equiv$  Population of the  $k^{\text{th}}$  state,  $P \equiv$  Total Population,  $D \equiv$  Standard Divisor  
 $Q_k \equiv$  Quota of the  $k^{\text{th}}$  state,  $A_k \equiv$  Apportionment of the  $k^{\text{th}}$  state

- Hamilton's Method:

STEP 1: Compute **standard divisor**

$$D = \frac{P}{M}$$

STEP 2: Compute quotas, **always rounding down**

$$Q_k = \left\lfloor \frac{P_k}{D} \right\rfloor$$

STEP 3: Compute the **total quota**

$$T = \sum_{k=1}^N Q_k$$

STEP 4: If  $T < M$ , assign each of the  $(M - T)$  surplus seats (one at a time) to the states having **quotas** with the **largest fractional parts**

$$A_k = Q_k + (\textit{surplus})$$

- Alabama Paradox: Increasing the total seats may **decrease** a state's apportionment.
- New States Paradox: The addition of a new state with its fair share of seats can affect apportionment of other states.

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### PIRNOT 10.3:

- Jefferson's Method:

STEP 1: Compute **standard divisor**

$$D = \frac{P}{M}$$

STEP 2: Compute divisor

$$D^* = D \left[ 1 + \alpha \left( \frac{N}{M} \right) \right]$$

STEP 3: Compute quotas, **rounding down**

$$Q_k = \left\lfloor \frac{P_k}{D^*} \right\rfloor$$

STEP 4: The apportionment is precisely the quota  $A_k = Q_k$

- Adams' Method:

STEP 1: Compute **standard divisor**

$$D = \frac{P}{M}$$

STEP 2: Compute divisor

$$D^* = D \left[ 1 + \alpha \left( \frac{N}{M} \right) \right]$$

STEP 3: Compute quotas, **rounding up**

$$Q_k = \left\lceil \frac{P_k}{D^*} \right\rceil$$

STEP 4: The apportionment is precisely the quota  $A_k = Q_k$

- Webster's Method:

STEP 1: Compute **standard divisor**

$$D = \frac{P}{M}$$

STEP 2: Compute divisor

$$D^* = D \left[ 1 + \alpha \left( \frac{N}{M} \right) \right]$$

STEP 3: Compute quotas, **rounding as usual**

$$Q_k = \left\lfloor \frac{P_k}{D^*} \right\rfloor$$

STEP 4: The apportionment is precisely the quota  $A_k = Q_k$

### PIRNOT 11.1:

- Plurality Method:

SETUP: Single-Winner Election has  $k$  candidates  
PROCESS: Each voter votes for one candidate  
WINNER: Candidate receiving the **most votes**

- Borda Count Method:

SETUP: Single-Winner Election has  $k$  candidates  
PROCESS: (1) Each voter ranks all candidates as follows:  
The 1<sup>st</sup> choice receives  $k$  points  
The 2<sup>nd</sup> choice receives  $(k - 1)$  points  
The 3<sup>rd</sup> choice receives  $(k - 2)$  points  
⋮  
The last choice receives 1 point  
(2) For each candidate, compute the total sum of points  
WINNER: Candidate receiving the **most total points**

- Plurality-with-Elimination Method:

SETUP: Single-Winner Election has  $k$  candidates  
PROCESS: (0) Compute total votes & # votes needed for a majority =  $\left\lceil \frac{(\text{total votes})}{2} \right\rceil$   
(1) If no candidate receives a majority of votes, then drop candidate(s) with fewest votes from the ballot  
(2) Conduct a new election round with updated ballot  
Assume voters don't change their preferences each round  
(3) Repeat (1)-(2) until a candidate receives a majority  
WINNER: Candidate receiving a **majority of votes**

- Pairwise Comparison Method:

SETUP: Single-Winner Election has  $k$  candidates  
PROCESS: (1) Voters rank all candidates  
(2) Pit candidates A and B "head-to-head"  
Count how many voters prefer A to B  
Count how many voters prefer B to A  
If A and B are tied, then each receives 1/2 point  
Else the more preferred candidate receives 1 point  
and the less preferred candidate receives 0 points  
(3) Repeat Step (2) for each pair of candidates  
WINNER: Candidate receiving the **most points**

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### PIRNOT 11.2:

- Majority Criterion:

If a majority of the voters rank a candidate as their 1<sup>st</sup> choice, then that candidate should win the election.

- Condorcet Criterion:

If candidate X can defeat each of the other candidates head-to-head, then candidate X is the winner of the election.

- IIA Criterion:

If candidate X wins, some nonwinner(s) are removed from ballot, and a recount is done, then candidate X still wins.

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### PIRNOT 11.3:

- A **weighted voting system** with  $N$  voters is described by the following:

$$[(\text{quota}) : (\text{weight of voter 1}), (\text{weight of voter 2}), \dots, (\text{weight of voter } N)] \equiv [Q : w_1, w_2, \dots, w_N]$$

The **quota**  $Q$  is the # of votes needed in this system to get a **motion** or **resolution** (i.e. vote "Yes" or "No") passed.

The **weights**  $w_1, w_2, \dots, w_N$  are the amount of votes controlled by voter 1, voter 2, ..., voter  $N$

- A **coalition** is a group of voters who vote the same way.

A **coalition's weight**  $W$  is the sum of the weights of all voters in the coalition:  $W = \sum_{k=1}^N w_k$

A coalition is called a **winning coalition** if the coalition's weight is greater than or equal to the quota:  $W \geq Q$

Voter  $k$  in a coalition is a **dictator** if voter  $k$  has total control:  $w_k \geq Q$

Voter  $k$  in a winning coalition is **critical** if the coalition needs voter  $k$  to win:  $W - w_k < Q$

- (Banzhaf Power for Voter  $k$ ) =  $\frac{\# \text{ times voter } k \text{ is } \mathbf{critical} \text{ in winning coalitions}}{\text{Total } \# \text{ times voters are } \mathbf{critical} \text{ in winning coalitions}} : B_k = \frac{C_k}{T}$ , where  $T = \sum_{k=1}^N C_k$

**PIRNOT 3.1:**

• Logic Connectives:

CONNECTIVE NAME:	NOTATION:	MEANING:
Conjunction	$P \wedge Q$	$P$ and $Q$
Disjunction	$P \vee Q$	$P$ or $Q$
Negation	$\sim P$	not $P$
Conditional	$P \rightarrow Q$	if $P$ then $Q$
Biconditional	$P \leftrightarrow Q$	$P$ if and only if $Q$

• Quantifiers:

**Universal Quantifiers:** "All", "Every", "Each"

**Existential Quantifiers:** "Some", "At least one", "There exists", "There is/are"

**PIRNOT 3.2:**

• Truth Tables for Connectives:

$P$	$Q$	$P \wedge Q$	$P \vee Q$	$\sim P$	$P \rightarrow Q$	$P \leftrightarrow Q$
T	T	T	T	F	T	T
T	F	F	T	F	F	F
F	T	F	T	T	T	F
F	F	F	F	T	T	T

• Logical Connectives (Order of Operations):

DOMINANCE:	CONNECTIVES:
MOST DOMINANT	Biconditional $\leftrightarrow$
$2^{nd}$ DOMINANT	Conditional $\rightarrow$
$3^{rd}$ DOMINANT	Conjunction $\wedge$ Disjunction $\vee$
LEAST DOMINANT	Negation $\sim$

• Logical Equivalence:

Two logic statements are **logically equivalent**, if they have the **same variables** (e.g.  $P, Q, R, \dots$ ) and the **final columns** in their **truth tables** are **identical**.

• DeMorgan's Laws:  $\sim(\sim P) \iff P$ ,  $\sim(P \wedge Q) \iff (\sim P) \vee (\sim Q)$ ,  $\sim(P \vee Q) \iff (\sim P) \wedge (\sim Q)$

**PIRNOT 3.3:**

• Truth Tables for Conditionals & Biconditionals: (see PIRNOT 3.1 above)

• More about Conditionals:

"If  $P$ , then  $Q$ "  $\iff$  " $Q$  if  $P$ "  $\iff$  " $P$  only if  $Q$ "  $\iff$  " $P$  is sufficient for  $Q$ "  $\iff$  " $Q$  is necessary for  $P$ "

• Converses, Inverses, Contrapositives of Conditionals:

The <b>converse</b> of conditional $P \rightarrow Q$ is	$Q \rightarrow P$
The <b>inverse</b> of conditional $P \rightarrow Q$ is	$\sim P \rightarrow \sim Q$
The <b>contrapositive</b> of conditional $P \rightarrow Q$ is	$\sim Q \rightarrow \sim P$

**PIRNOT 3.4:**

• Common Arguments & Fallacies:

Law of Detachment:	$P \rightarrow Q, P$	$\models Q$
Law of Contraposition:	$P \rightarrow Q, \sim Q$	$\models \sim P$
Law of Syllogism:	$P \rightarrow Q, Q \rightarrow R$	$\models P \rightarrow R$
Disjunctive Syllogism:	$P \vee Q, \sim P$	$\models Q$
Fallacy of the Converse:	$P \rightarrow Q, Q$	$\not\models P$
Fallacy of the Inverse:	$P \rightarrow Q, \sim P$	$\not\models \sim Q$
Affirming a Disjunction:	$P \vee Q, P$	$\not\models \sim Q$

#### PIRNOT 4.1:

- Basics:

A **graph** consists of a finite set of points, called **vertices**, and lines/curves, called **edges**, that join pairs of vertices.

An **isolated vertex** has no edges joined to it. A **loop** is an edge which joins one vertex with itself.

A graph is **connected** if it's possible to travel from any vertex to any other vertex by moving along successive edges.

An edge of a connected graph is a **bridge** if removing the edge causes the graph to no longer be connected.

- Degree of a Vertex:

The **degree** of a vertex is the # of edges joined to that vertex.

**Loops** count as **two edges**. The degree of an **isolated vertex** is defined to be **zero**.

An **odd vertex** is a vertex with an odd degree. An **even vertex** is a vertex with an even degree.

- Euler Paths & Euler Circuits:

A **path** is a sequence of consecutive edges in which no edge is repeated. The **length** of a path is # of edges in the path.

An **Euler path** is a path that contains all edges of the graph. An **Euler circuit** is an Euler path that begins & ends at same vertex.

- Euler's Theorem:

A graph can be traced if it's connected and has zero or two odd vertices.

A graph is traceable if it contains an Euler path or Euler circuit.

(a) Suppose a graph has two odd vertices.

Then, the tracing must begin at one odd vertex and end at the other.

(b) Suppose a graph has zero odd vertices. (i.e. all vertices are even)

Then, the tracing must begin and end at the same vertex.

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#### PIRNOT 4.2:

- Hamilton Paths & Hamilton Circuits: A **Hamilton path** is a path that passes through all vertices exactly once.

A **Hamilton circuit** is a Hamilton path that begins & ends at the same vertex.

- Finding an Optimal Hamilton Circuit via Brute Force Algorithm:

INPUT: A connected weighted graph.

(1) List all Hamilton circuits in the graph.

(2) Find the weight of each circuit found in (1).

OUTPUT: Optimal Hamilton circuit(s) with the smallest weights

- Finding a Semi-Optimal Hamilton Circuit via Nearest Neighbor Algorithm:

INPUT: A connected weighted graph.

(1) Start at any vertex.

(2) Choose the edge joined to the vertex with the smallest weight.

Traverse the chosen edge.

(3) Repeat (2) until all vertices have been touched.

(4) Close the circuit by returning to the starting vertex.

OUTPUT: Semi-optimal Hamilton circuit with "nearly" smallest weight.

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#### PIRNOT 4.3:

- Directed Edges, Directed Graphs, Directed Paths:

A **directed edge** is an edge with a direction. A **directed graph** is a graph in which all edges are directed.

A **directed path** from vertex  $X$  to vertex  $Y$  in a directed graph is a sequence of edges starting at  $X$ , following the edges in the prescribed directions, and ending at  $Y$ .

The **length** of a directed path is the # of edges along that path. The length of a **directed loop** is one (not two).

### PIRNOT 13.1:

$S \equiv$  Sample Space,  $E, F \equiv$  Events

- Measure of a Set:

The **measure** of a **countable set** is defined as:  $m(E) = (\# \text{ of elements in } E)$

The **measure** of a **1D set** is defined as:  $m(E) = (\text{Length of } E)$

The **measure** of a **2D set** is defined as:  $m(E) = (\text{Area of } E)$

The **measure** of a **3D set** is defined as:  $m(E) = (\text{Volume of } E)$

The **measure** of the **empty set** is defined to be zero:  $m(\emptyset) = 0$

- Probability of an Event Occurring:  $P(E) = \frac{m(E)}{m(S)}$

- Properties of Probability: (a)  $0 \leq P(E) \leq 1$  (b)  $P(\emptyset) = 0$  (c)  $P(S) = 1$

- Complement of an Event: The **complement** of event  $E$ , denoted  $E^c$ , is the set of all outcomes in  $S$  that are not in  $E$ .

REMARK: The complement of the sample space is the empty set:  $S^c = \emptyset$

REMARK: The complement of the empty set is the sample space:  $\emptyset^c = S$

REMARK: The textbook denotes the complement of  $E$  as  $E'$ .

- Odds in Favor of an Event: (**Odds in favor** of event  $E$ ) =  $\frac{P(E)}{P(E^c)}$

- Odds Against an Event: (**Odds against** an event  $E$ ) =  $\frac{P(E^c)}{P(E)}$

### PIRNOT 13.2:

- Probability of an Event Not Occurring:  $P(\text{Not } E) = P(E^c) = 1 - P(E)$

- Probability of a Disjunction of Two Events:  $P(E \text{ or } F) = P(E \cup F) = P(E) + P(F) - P(E \cap F)$

- Mutually Exclusive Events: Two events  $E, F$  are **mutually exclusive** if they have no outcomes in common:

$$E \cap F = \emptyset \iff P(E \text{ and } F) = P(E \cap F) = 0$$

- Probability of Two Events Not Occurring:  $P(\text{Neither } E \text{ nor } F) = P[(E \cup F)^c] = 1 - P(E) - P(F) + P(E \cap F)$

### PIRNOT 13.3:

- Conditional Probability:  $P(\text{If } E \text{ then } F) = P(F \text{ given } E) = P(F|E) = \frac{P(E \cap F)}{P(E)}$

- Intersection of Events (Alternative):  $P(E \cap F) = P(E) \cdot P(F|E)$  or equivalently  $P(E \cap F) = P(F) \cdot P(E|F)$

- Independence of Events: Events  $E$  and  $F$  are **independent** if:

$$P(F|E) = P(F) \text{ or equivalently } P(E|F) = P(E) \text{ or equivalently } P(E \cap F) = P(E) \cdot P(F)$$

Otherwise, events  $E$  and  $F$  are **dependent**.

### PIRNOT 13.4:

- Expected Value: Suppose an experiment has a sample space with  $N$  possible outcomes with probabilities  $P_1, P_2, \dots, P_N$ .

Moreover, assume each outcome has an associated value with it that are labeled  $V_1, V_2, \dots, V_N$ .

Then, (Expected Value)  $\equiv EV = P_1V_1 + P_2V_2 + \dots + P_NV_N$

- Fairness of Games of Chance:

A game of chance is **fair** if the game has an **expected value of zero**:  $EV = 0$

A game of chance is **unfair** if it has an **expected value that's not zero**:  $EV \neq 0$

### **PIRNOT 14.1:**

Going forward, assume a data set contains  $n$  data values.

- Frequency Table: The **frequency** of a data category is the # data values in the data set that fall into that category.
  - Frequency Histogram: A **frequency histogram** is a bar graph with frequency as the **vertical axis**.
  - Stem-and-Leaf Display: A **stem-and-leaf display** is a table with each stem representing the 10's digit of the data values & each leaf representing the 1's digit of a data value.
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### **PIRNOT 14.2:**

- Mean of a Data Set: The **mean** is:  $\bar{x} = \frac{\sum x}{n}$
  - Mean of a Frequency Distribution: The **mean** is:  $\bar{x} = \frac{\sum(x \cdot f)}{\sum f}$
  - Median of a Data Set: The **median** is the middle value in the sorted data set.  
If there is an **odd** # of values, then the median is the value in the **middle position**.  
If there is an **even** # of values, then the median is the **average** of the **two middle values**.
  - Median of a Frequency Distribution: The **median** is the middle value in the sorted frequency distribution.  
If  $\sum f$  is **odd**, then the median is the value in the  $\left[ \frac{\sum f}{2} \right]$ -th position.  
If  $\sum f$  is **even**, then the median is the **average** of the values in the  $\left( \frac{\sum f}{2} \right)$ -th &  $\left[ \left( \frac{\sum f}{2} \right) + 1 \right]$ -st positions.
  - Mode of a Data Set: The **mode** is the data value that occurs most frequently.  
If two values occur most frequently, then each is a mode.  
If more than two values occur most frequently, then there is **no mode**.
  - Mode of a Frequency Distribution: The **mode** is the data value with the highest frequency.  
If two values occur most frequently, then each is a mode.  
If more than two values occur most frequently, then there is **no mode**.
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### **PIRNOT 14.3:**

- Range of a Data Set: The **range** is the difference between the largest and smallest data values.
  - Standard Deviation of a Data Set: The **standard deviation** is:  $s = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$
  - Range of a Frequency Distribution: The **range** is the difference between the largest and smallest data values.
  - Standard Deviation of a Frequency Distribution: The **standard deviation** is:  $s = \sqrt{\frac{\sum [(x - \bar{x})^2 \cdot f]}{(\sum f) - 1}}$
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### **PIRNOT 14.4:**

- Properties of Normal Distributions:  
A normal curve is bell-shaped.  
The highest point on a normal curve is at the mean of the distribution.  
The mean, median, and mode of the distribution are all the same value.  
The curve is symmetric with respect to its mean.
  - 68-95-99.7 Rule for Normal Distributions:  
Roughly 68% of the data values are within 1 standard deviation from the mean.  
Roughly 95% of the data values are within 2 standard deviations from the mean.  
Roughly 99.7% of the data values are within 3 standard deviation from the mean.
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