# WEIGHTED VOTING SYSTEMS [PIRNOT 11.3]

**EX 11.3.1:** Given the weighted voting system    $[10 : 5, 5, 4]$

(a) What is the quota?

(b) What are the weights of the voters?

(c) Are there any dictators?

(d) Which voters, if any, have veto power?

---

**EX 11.3.2:** Given the weighted voting system    $[3 : 4, 1, 1, 1]$

(a) What is the quota?

(b) What are the weights of the voters?

(c) Are there any dictators?

(d) Which voters, if any, have veto power?

Given the weighted voting system    $[10 : 5, 5, 4]$

where Voter 1 is a (D)emocrat, Voter 2 is a (R)epublican, Voter 3 is a (T)hird-party.

(a) Complete the following table of all possible coalitions:

| COALITION | WEIGHT | QUOTA | WINNING? | CRITICAL VOTER(S) |
|---|---|---|---|---|
| $\{D\}$ | 5 | 10 | No | — |
| $\{R\}$ | 5 | 10 | No | — |
| $\{T\}$ | 4 | 10 | No | — |
| $\{D, R\}$ | 10 | 10 | Yes | $D, R$ |
| $\{D, T\}$ | 9 | 10 | No | — |
| $\{R, T\}$ | 9 | 10 | No | — |
| $\{D, R, T\}$ | 14 | 10 | Yes | $D, R$ |

(b) Compute the Banzhaf Power Index of each voter:

| VOTER | BANZHAF POWER INDEX |
|---|---|
| Voter 1 ($D$) | $\frac{2}{4} = \frac{1}{2}$ |
| Voter 2 ($R$) | $\frac{2}{4} = \frac{1}{2}$ |
| Voter 3 ($T$) | $\frac{0}{4} = 0$ |

A city council has 14 members with the following political party affiliations:

6 (D)emocrats, 4 (R)epublicans, 3 (I)ndependents, and 1 (L)ibertarian.

Resolutions are passed by a **simple majority**.

Assume that members **always vote along party lines**.

Moreover, assume the 3 Independents are part of the Independent Party.

(a) Write down the representative weighted voting system (put the **weights** in **descending order**).

(b) Complete the following table of all possible coalitions:

| COALITION | WEIGHT | QUOTA | WINNING? | CRITICAL VOTER(S) |
|---|---|---|---|---|
| $\{D\}$ | | | | |
| $\{R\}$ | | | | |
| $\{I\}$ | | | | |
| $\{L\}$ | | | | |
| $\{D, R\}$ | | | | |
| $\{D, I\}$ | | | | |
| $\{D, L\}$ | | | | |
| $\{R, I\}$ | | | | |
| $\{R, L\}$ | | | | |
| $\{I, L\}$ | | | | |
| $\{D, R, I\}$ | | | | |
| $\{D, R, L\}$ | | | | |
| $\{D, I, L\}$ | | | | |
| $\{R, I, L\}$ | | | | |
| $\{D, R, I, L\}$ | | | | |

(c) Compute the Banzhaf Power Index of each voter:

| VOTER | BANZHAF POWER INDEX |
|---|---|
| Democratic Party $(D)$ | |
| Republican Party $(R)$ | |
| Independent Party $(I)$ | |
| Libertarian Party $(L)$ | |