CONDITIONAL PROBABILITY & INDEPENDENCE OF EVENTS [PIRNOT 13.3]

EX 13.3.1: One fair 3-sided die & one fair 4-sided die are both rolled.

(a) Determine the sample space for the experiment.

The sample space is the **set** of **all possible outcomes** for the experiment:

$$\boxed{S = \{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(2,4),(3,1),(3,2),(3,3),(3,4)\}}$$

(b) Find $P(\text{die 1 shows 3})$, $P(\text{die 2 shows 4})$, $P(\text{die 1 shows 3 and die 2 shows 4})$

Let events $E_1 \equiv \text{Die 1 shows 3} = \{(3,1),(3,2),(3,3),(3,4)\}$

$E_2 \equiv \text{Die 2 shows 4} = \{(1,4),(2,4),(3,4)\}$

Then event $E_1 \cap E_2 \equiv$ Die 1 shows 3 **and** Die 2 shows 4 = (All outcomes **common** to both E_1 **and** E_2) $= \{(3,4)\}$

$$\Longrightarrow P(E_1) = \frac{m(E_1)}{m(S)} = \frac{(\#\text{ outcomes in event } E_1)}{(\#\text{ outcomes in sample space } S)} = \frac{4}{12} = \boxed{\frac{1}{3}}$$

$$\Longrightarrow P(E_2) = \frac{m(E_2)}{m(S)} = \frac{(\#\text{ outcomes in event } E_2)}{(\#\text{ outcomes in sample space } S)} = \frac{3}{12} = \boxed{\frac{1}{4}}$$

$$\Longrightarrow P(E_1 \cap E_2) = \frac{m(E_1 \cap E_2)}{m(S)} = \frac{(\#\text{ outcomes in event } E_1 \cap E_2)}{(\#\text{ outcomes in sample space } S)} = \boxed{\frac{1}{12}}$$

(c) Find the probability that die 1 shows 3 given die 2 shows 4.

$$P(E_1 \text{ given } E_2) = P(E_1|E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)} = \frac{1/12}{1/4} = \frac{1}{12} \div \frac{1}{4} = \frac{1}{12} \times \frac{4}{1} = \frac{4}{12} = \boxed{\frac{1}{3}}$$

(d) Find the probability that die 2 shows 4 given die 1 shows 3.

$$P(E_2 \text{ given } E_1) = P(E_2|E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)} = \frac{1/12}{1/3} = \frac{1}{12} \div \frac{1}{3} = \frac{1}{12} \times \frac{3}{1} = \frac{3}{12} = \boxed{\frac{1}{4}}$$

(e) Are the events "die 1 shows 3" & "die 2 shows 4" independent?

There are three ways to answer this:

1^{st} way: $P(E_1|E_2) = \frac{1}{3}$ and $P(E_1) = \frac{1}{3} \Longrightarrow P(E_1|E_2) = P(E_1) \Longrightarrow$ Events E_1 and E_2 are **Independent**

2^{nd} way: $P(E_2|E_1) = \frac{1}{4}$ and $P(E_2) = \frac{1}{4} \Longrightarrow P(E_2|E_1) = P(E_2) \Longrightarrow$ Events E_1 and E_2 are **Independent**

3^{rd} way: $P(E_1)P(E_2) = \frac{1}{3} \times \frac{1}{4} = \frac{1}{12} = P(E_1 \cap E_2) \Longrightarrow$ Events E_1 and E_2 are **Independent**