LOGIC: TRUTH TABLES, LOGICAL EQUIV, DEMORGAN'S LAWS [PIRNOT 3.2]

EX 3.2.3: Using truth tables, determine whether these two statements are logically equivalent: $P \wedge (\sim Q)$, $\sim (\sim P \vee Q)$

Construct a single truth table containing the variables, intermediate expressions, and the two desired expressions:

P	Q	$\sim Q$	$P \wedge (\sim Q)$	$\sim P$	$\sim P \vee Q$	$\sim (\sim P \vee Q)$
T	T	F	**F**	F	T	**F**
T	F	T	**T**	F	F	**T**
F	T	F	**F**	T	T	**F**
F	F	T	**F**	T	T	**F**

NOTE: Having and filling out the columns for the intermediate expressions [$\sim Q$, $\sim P$, $\sim P \vee Q$] are **absolutely critical** for **painlessly** finding the two desired expressions [$P \wedge (\sim Q)$, $\sim (\sim P \vee Q)$].

Since the columns for $P \wedge (\sim Q)$ and $\sim (\sim P \vee Q)$ (in **bold**) are **identical**, these two expressions are logically equivalent.
One can indicate the two expressions are logically equivalent symbolically as follows:

$$P \wedge (\sim Q) \iff \sim (\sim P \vee Q)$$

EX 3.2.6: Using DeMorgan's Laws, negate the following English statement (in English): "Cats meow and dogs bark."

Let logic statements $P \equiv$ "Cats meow", $Q \equiv$ "Dogs bark"

Then translate given English statement to a symbolic logic statement:

"Cats meow and dogs bark" $\equiv P \wedge Q$

Now apply appropriate DeMorgan's Law(s) on the negation of $P \wedge Q$:

$$\sim (P \wedge Q) \iff (\sim P) \vee (\sim Q)$$

Finally, translate the simplified symbolic logic statement to English:

$(\sim P) \vee (\sim Q) \equiv$ "Cats do not meow or dogs do not bark."

NOTE: The symbol $\equiv$ means "represents" (and is **not** a logical connective!)

NOTE: The symbol $\iff$ means "is logically equivalent to" (and is **not** a logical connective!)
