Voting Methods

Contemporary Math

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Introduction

In free societies, citizens **vote** for politicians whose values & opinions on contemporary issues align with theirs.

Unfortunately, voting is not as clear-cut as one would expect.
Consider U.S. presidential elections:

- 2000 Face-Off [Al Gore (D) vs George W. Bush (R)]:
  - Gore received more (popular) votes than Bush, yet Bush won!
  - Bush received more **electoral votes** and carried more states than Gore.
- 2012 Florida (R) Primary [Romney vs Gingrich vs Paul vs Santorum]:
  - Mitt Romney earned all 50 delegates yet he earned $< 50\%$ of the votes.
  - Newt Gingrich earned no delegates yet he earned 32% of the votes.
- After a few primaries, most candidates drop out before most citizens have a chance to vote due to lack of funding.
- Winning depends not only on vote counts but also on how the votes are used!

Because of these issues, several voting methods exist.

Preference Ballots & Tables

Suppose there are five candidates: A,B,C,D,E

Each voter uses a **preference ballot** to rank the candidates:

1^{st}	D
2^{nd}	B
3^{rd}	E
4^{th}	A
5^{th}	C

Identical preference ballots are grouped together in a **preference table**:

	Number of Ballots				
Preference	8	3	6	7	2
1^{st}	A	E	B	A	D
2^{nd}	B	D	D	C	B
3^{rd}	C	A	C	E	E
4^{th}	D	B	E	B	A
5^{th}	E	C	A	D	C

Plurality Method (Definition)

The simplest, most intuitive voting method is the **Plurality Method**:

Definition

(Plurality Method)

SETUP:	Single-Winner Election has k candidates
PROCESS:	Each voter votes for one candidate
WINNER:	Candidate receiving the **most votes**

Plurality Method is typically used by:

- State elections
- Local elections
  - City council elections
  - School board elections

Plurality Method (Example)

WEX 11-1-1: Using Plurality Method, determine election winner:

	Number of Ballots				
Preference	10	7	5	5	4
1^{st} choice	A	D	B	C	B
2^{nd} choice	C	B	C	D	C
3^{rd} choice	B	A	A	A	D
4^{th} choice	D	C	D	B	A

Plurality Method (Example)

WEX 11-1-1: Using Plurality Method, determine election winner:

	Number of Ballots				
Preference	10	7	5	5	4
1^{st} choice	A	D	B	C	B
2^{nd} choice	C	B	C	D	C
3^{rd} choice	B	A	A	A	D
4^{th} choice	D	C	D	B	A

With Plurality Method, only the first choice matters (in blue)

Plurality Method (Example)

WEX 11-1-1: Using Plurality Method, determine election winner:

	Number of Ballots				
Preference	10	7	5	5	4
1^{st} choice	A	D	B	C	B
2^{nd} choice	C	B	C	D	C
3^{rd} choice	B	A	A	A	D
4^{th} choice	D	C	D	B	A

With Plurality Method, only the first choice matters (in blue)

A has 10 votes

B has $5 + 4 = 9$ votes

C has 5 votes

D has 7 votes

Plurality Method (Example)

WEX 11-1-1: Using Plurality Method, determine election winner:

	Number of Ballots				
Preference	10	7	5	5	4
1^{st} choice	*A*	*D*	*B*	*C*	*B*
2^{nd} choice	C	B	C	D	C
3^{rd} choice	B	A	A	A	D
4^{th} choice	D	C	D	B	A

With Plurality Method, only the first choice matters (in blue)

A has 10 votes

B has $5 + 4 = 9$ votes

C has 5 votes

D has 7 votes

Since A has the most votes, A is the winner

Borda Count Method (Definition)

What if, instead, voters must **rank** each candidate?

Definition

(Borda Count Method)

SETUP: Single-Winner Election has k candidates

PROCESS: (1) Each voter ranks all candidates as follows:

The 1^{st} choice receives k points

The 2^{nd} choice receives $(k-1)$ points

The 3^{rd} choice receives $(k-2)$ points

$\vdots$

The last choice receives 1 point

(2) For each candidate, compute the total sum of points

WINNER: Candidate receiving the **most total points**

Borda Count Method is typically used in:

- sports polls, music industry awards, hiring CEO's, ...

Borda Count Method (Example)

WEX 11-1-2: Using Borda Count Method, determine election winner:

	Number of Ballots				
Preference	10	7	5	5	4
1^{st} choice	A	D	B	C	B
2^{nd} choice	C	B	C	D	C
3^{rd} choice	B	A	A	A	D
4^{th} choice	D	C	D	B	A

Borda Count Method (Example)

WEX 11-1-2: Using Borda Count Method, determine election winner:

	Number of Ballots				
Preference	10	7	5	5	4
1^{st} choice/4	A	D	B	C	B
2^{nd} choice/3	C	B	C	D	C
3^{rd} choice/2	B	A	A	A	D
4^{th} choice/1	D	C	D	B	A

Assign points to each preference catagory (in blue)

Borda Count Method (Example)

WEX 11-1-2: Using Borda Count Method, determine election winner:

	Number of Ballots				
Preference	10	7	5	5	4
1^{st} choice/4	A/(4×10)	D/(4×7)	B/(4×5)	C/(4×5)	B/(4×4)
2^{nd} choice/3	C/(3×10)	B/(3×7)	C/(3×5)	D/(3×5)	C/(3×4)
3^{rd} choice/2	B/(2×10)	A/(2×7)	A/(2×5)	A/(2×5)	D/(2×4)
4^{th} choice/1	D/(1×10)	C/(1×7)	D/(1×5)	B/(1×5)	A/(1×4)

Assign points to each choice

Borda Count Method (Example)

WEX 11-1-2: Using Borda Count Method, determine election winner:

	Number of Ballots				
Preference	10	7	5	5	4
1^{st} choice/4	A/40	D/28	B/20	C/20	B/16
2^{nd} choice/3	C/30	B/21	C/15	D/15	C/12
3^{rd} choice/2	B/20	A/14	A/10	A/10	D/8
4^{th} choice/1	D/10	C/7	D/5	B/5	A/4

Assign points to each choice (and simplify)

Borda Count Method (Example)

WEX 11-1-2: Using Borda Count Method, determine election winner:

	Number of Ballots				
Preference	10	7	5	5	4
1^{st} choice/4	$A/40$	D/28	B/20	C/20	B/16
2^{nd} choice/3	C/30	B/21	C/15	D/15	C/12
3^{rd} choice/2	B/20	$A/14$	$A/10$	$A/10$	D/8
4^{th} choice/1	D/10	C/7	D/5	B/5	$A/4$

Tally points for each candidate:

A: $40 + 14 + 10 + 10 + 4 = 78$ points

Borda Count Method (Example)

WEX 11-1-2: Using Borda Count Method, determine election winner:

	Number of Ballots				
Preference	10	7	5	5	4
1^{st} choice/4	A/40	D/28	*B*/20	C/20	*B*/16
2^{nd} choice/3	C/30	*B*/21	C/15	D/15	C/12
3^{rd} choice/2	*B*/20	A/14	A/10	A/10	D/8
4^{th} choice/1	D/10	C/7	D/5	*B*/5	A/4

Tally points for each candidate:

A: $40 + 14 + 10 + 10 + 4 = 78$ points

B : $20 + 21 + 20 + 5 + 16 = 82$ points

Borda Count Method (Example)

WEX 11-1-2: Using Borda Count Method, determine election winner:

	Number of Ballots				
Preference	10	7	5	5	4
1^{st} choice/4	A/40	D/28	B/20	C/20	B/16
2^{nd} choice/3	C/30	B/21	C/15	D/15	C/12
3^{rd} choice/2	B/20	A/14	A/10	A/10	D/8
4^{th} choice/1	D/10	C/7	D/5	B/5	A/4

Tally points for each candidate:

A: $40 + 14 + 10 + 10 + 4 = 78$ points

B: $20 + 21 + 20 + 5 + 16 = 82$ points

C: $30 + 7 + 15 + 20 + 12 = 84$ points

Borda Count Method (Example)

WEX 11-1-2: Using Borda Count Method, determine election winner:

	Number of Ballots				
Preference	10	7	5	5	4
1^{st} choice/4	A/40	$D/28$	B/20	C/20	B/16
2^{nd} choice/3	C/30	B/21	C/15	$D/15$	C/12
3^{rd} choice/2	B/20	A/14	A/10	A/10	$D/8$
4^{th} choice/1	$D/10$	C/7	$D/5$	B/5	A/4

Tally points for each candidate:

A: $40 + 14 + 10 + 10 + 4 = 78$ points

B: $20 + 21 + 20 + 5 + 16 = 82$ points

C: $30 + 7 + 15 + 20 + 12 = 84$ points

D: $10 + 28 + 5 + 15 + 8 = 52$ points

Borda Count Method (Example)

WEX 11-1-2: Using Borda Count Method, determine election winner:

	Number of Ballots				
Preference	10	7	5	5	4
1^{st} choice/4	A/40	D/28	B/20	C/20	B/16
2^{nd} choice/3	C/30	B/21	C/15	D/15	C/12
3^{rd} choice/2	B/20	A/14	A/10	A/10	D/8
4^{th} choice/1	D/10	C/7	D/5	B/5	A/4

Tally points for each candidate:

A: $40 + 14 + 10 + 10 + 4 = 78$ points

B: $20 + 21 + 20 + 5 + 16 = 82$ points

C: $30 + 7 + 15 + 20 + 12 = 84$ points

D: $10 + 28 + 5 + 15 + 8 = 52$ points

Since Candidate C has the most points, C is the winner

Plurality-with-Elimination Method (Definition)

Definition

(Plurality-with-Elimination Method)

SETUP:		Single-Winner Election has k candidates
PROCESS:	(0)	Compute total votes & # votes needed for a majority
	(1)	If no candidate receives a majority of votes, then drop candidate(s) with fewest votes from the ballot
	(2)	Conduct a new election round with updated ballot Assume voters don't change their preferences each round*
	(3)	Repeat (1)-(2) until a candidate receives a majority
WINNER:		Candidate receiving a **majority of votes**

*If voters prefer A to B and B to C, then if B's dropped, voters will prefer A to C.

Plurality-with-Elimination Method is typically used in:

- Municipal Elections (e.g. city mayor)

Plurality-with-Elimination Method (Example)

WEX 11-1-3: Using Plurality-with-Elimination Mtd, determine election winner:

	Number of Ballots				
Preference	10	7	5	5	4
1^{st} choice	A	D	B	C	B
2^{nd} choice	C	B	C	D	C
3^{rd} choice	B	A	A	A	D
4^{th} choice	D	C	D	B	A

Plurality-with-Elimination Method (Example)

WEX 11-1-3: Using Plurality-with-Elimination Mtd, determine election winner:

	Number of Ballots				
Preference	10	7	5	5	4
1^{st} choice	A	D	B	C	B
2^{nd} choice	C	B	C	D	C
3^{rd} choice	B	A	A	A	D
4^{th} choice	D	C	D	B	A

First of all, compute the **total votes** & **votes needed for majority**:

Total votes = 10 + 7 + 5 + 5 + 4 = 31

31/2 = 15.5 $\Longrightarrow$ At least 16 votes needed for a majority

Plurality-with-Elimination Method (Example)

WEX 11-1-3: Using Plurality-with-Elimination Mtd, determine election winner:

	Number of Ballots				
Preference	10	7	5	5	4
1^{st} choice	A	D	B	C	B
2^{nd} choice	C	B	C	D	C
3^{rd} choice	B	A	A	A	D
4^{th} choice	D	C	D	B	A

Total votes = 31 At least 16 votes needed for a majority

Round 1

Plurality-with-Elimination Method (Example)

WEX 11-1-3: Using Plurality-with-Elimination Mtd, determine election winner:

	Number of Ballots				
Preference	10	7	5	5	4
1^{st} choice	*A*	*D*	*B*	*C*	*B*
2^{nd} choice	C	B	C	D	C
3^{rd} choice	B	A	A	A	D
4^{th} choice	D	C	D	B	A

Total votes = 31 At least 16 votes needed for a majority

Count 1^{st}-choice votes for each candidate:

$$
\begin{aligned}
A &= 10 < 16 \implies (\text{NOT a majority}) \\
B &= 5+4=9 < 16 \implies (\text{NOT a majority}) \\
C &= 5 < 16 \implies (\text{NOT a majority}) \\
D &= 7 < 16 \implies (\text{NOT a majority})
\end{aligned}
$$

Plurality-with-Elimination Method (Example)

WEX 11-1-3: Using Plurality-with-Elimination Mtd, determine election winner:

	Number of Ballots				
Preference	10	7	5	5	4
1^{st} choice	A	D	B	C	B
2^{nd} choice	C	B	C	D	C
3^{rd} choice	B	A	A	A	D
4^{th} choice	D	C	D	B	A

Total votes = 31 At least 16 votes needed for a majority

Count 1^{st}-choice votes for each candidate:

$$A = 10 < 16 \implies (\text{NOT a majority})$$
$$B = 5 + 4 = 9 < 16 \implies (\text{NOT a majority})$$
$$C = 5 < 16 \implies (\text{NOT a majority})$$
$$D = 7 < 16 \implies (\text{NOT a majority})$$

Since no candidate has a majority, eliminate candidate(s) with fewest votes: C

Plurality-with-Elimination Method (Example)

WEX 11-1-3: Using Plurality-with-Elimination Mtd, determine election winner:

	Number of Ballots				
Preference	10	7	5	5	4
1^{st} choice	A	D	B	*C*	B
2^{nd} choice	*C*	B	*C*	D	*C*
3^{rd} choice	B	A	A	A	D
4^{th} choice	D	*C*	D	B	A

Total votes = 31 At least 16 votes needed for a majority

Since no candidate has a majority, eliminate candidate(s) with fewest votes: C

Plurality-with-Elimination Method (Example)

WEX 11-1-3: Using Plurality-with-Elimination Mtd, determine election winner:

	Number of Ballots				
Preference	10	7	5	5	4
1^{st} choice	A	D	B	*C*	B
2^{nd} choice	*C*	B	*C*	*D*	*C*
3^{rd} choice	*B*	A	*A*	*A*	*D*
4^{th} choice	*D*	*C*	*D*	*B*	*A*

Total votes = 31 At least 16 votes needed for a majority

Eliminate candidate C, moving every entry below C (in blue) up one row

Plurality-with-Elimination Method (Example)

WEX 11-1-3: Using Plurality-with-Elimination Mtd, determine election winner:

	Number of Ballots				
Preference	10	7	5	5	4
1^{st} choice	A	D	B	*D*	B
2^{nd} choice	*B*	B	*A*	*A*	*D*
3^{rd} choice	*D*	A	*D*	*B*	*A*
4^{th} choice					

Total votes = 31 At least 16 votes needed for a majority

Eliminate candidate C, moving every entry below C (in blue) up one row

Plurality-with-Elimination Method (Example)

WEX 11-1-3: Using Plurality-with-Elimination Mtd, determine election winner:

	Number of Ballots				
Preference	10	7	5	5	4
1^{st} choice	A	D	B	D	B
2^{nd} choice	B	B	A	A	D
3^{rd} choice	D	A	D	B	A

Total votes = 31 At least 16 votes needed for a majority

Round 2

Plurality-with-Elimination Method (Example)

WEX 11-1-3: Using Plurality-with-Elimination Mtd, determine election winner:

	Number of Ballots				
Preference	10	7	5	5	4
1^{st} choice	*A*	*D*	*B*	*D*	*B*
2^{nd} choice	B	B	A	A	D
3^{rd} choice	D	A	D	B	A

Total votes = 31 At least 16 votes needed for a majority

Count 1^{st}-choice votes for each candidate:

$$A = 10 < 16 \implies (\text{NOT a majority})$$
$$B = 5 + 4 = 9 < 16 \implies (\text{NOT a majority})$$
$$D = 7 + 5 = 12 < 16 \implies (\text{NOT a majority})$$

Plurality-with-Elimination Method (Example)

WEX 11-1-3: Using Plurality-with-Elimination Mtd, determine election winner:

	Number of Ballots				
Preference	10	7	5	5	4
1^{st} choice	A	D	*B*	D	*B*
2^{nd} choice	*B*	*B*	A	A	D
3^{rd} choice	D	A	D	*B*	A

Total votes = 31 At least 16 votes needed for a majority

Count 1^{st}-choice votes for each candidate:

$$A = 10 < 16 \implies (\text{NOT a majority})$$
$$B = 5 + 4 = 9 < 16 \implies (\text{NOT a majority})$$
$$D = 7 + 5 = 12 < 16 \implies (\text{NOT a majority})$$

Since no candidate has a majority, eliminate candidate(s) with fewest votes: B

Plurality-with-Elimination Method (Example)

WEX 11-1-3: Using Plurality-with-Elimination Mtd, determine election winner:

	Number of Ballots				
Preference	10	7	5	5	4
1^{st} choice	A	D	*B*	D	*B*
2^{nd} choice	*B*	*B*	A	A	D
3^{rd} choice	D	A	D	*B*	A

Total votes = 31 At least 16 votes needed for a majority

Since no candidate has a majority, eliminate candidate(s) with fewest votes: B

Plurality-with-Elimination Method (Example)

WEX 11-1-3: Using Plurality-with-Elimination Mtd, determine election winner:

	Number of Ballots				
Preference	10	7	5	5	4
1^{st} choice	A	D	*B*	D	*B*
2^{nd} choice	*B*	*B*	*A*	A	*D*
3^{rd} choice	*D*	*A*	*D*	*B*	*A*

Total votes = 31 At least 16 votes needed for a majority

Eliminate candidate B, moving every entry below B (in blue) up one row

Plurality-with-Elimination Method (Example)

WEX 11-1-3: Using Plurality-with-Elimination Mtd, determine election winner:

	Number of Ballots				
Preference	10	7	5	5	4
1^{st} choice	A	D	*A*	D	*D*
2^{nd} choice	*D*	*A*	*D*	A	*A*
3^{rd} choice					

Total votes = 31 At least 16 votes needed for a majority

Eliminate candidate B, moving every entry below B (in blue) up one row

Plurality-with-Elimination Method (Example)

WEX 11-1-3: Using Plurality-with-Elimination Mtd, determine election winner:

	Number of Ballots				
Preference	10	7	5	5	4
1^{st} choice	A	D	A	D	D
2^{nd} choice	D	A	D	A	A

Total votes = 31 At least 16 votes needed for a majority

Round 3

Plurality-with-Elimination Method (Example)

WEX 11-1-3: Using Plurality-with-Elimination Mtd, determine election winner:

	Number of Ballots				
Preference	10	7	5	5	4
1^{st} choice	A	D	A	D	D
2^{nd} choice	D	A	D	A	A

Total votes = 31 At least 16 votes needed for a majority

Count 1^{st}-choice votes for each candidate:

$$A = 10 + 5 = 15 < 16 \implies \text{(NOT a majority)}$$
$$D = 7 + 5 + 4 = 16 \geq 16 \implies \text{(MAJORITY!)}$$

Plurality-with-Elimination Method (Example)

WEX 11-1-3: Using Plurality-with-Elimination Mtd, determine election winner:

	Number of Ballots				
Preference	10	7	5	5	4
1^{st} choice	A	D	A	D	D
2^{nd} choice	D	A	D	A	A

Total votes = 31 At least 16 votes needed for a majority

Count 1^{st}-choice votes for each candidate:

$$A = 10 + 5 = 15 < 16 \implies \text{(NOT a majority)}$$
$$D = 7 + 5 + 4 = 16 \geq 16 \implies \text{(MAJORITY!)}$$

Since candidate D has a majority, D is the winner

Pairwise Comparison Method (Definition)

The election winner is expected to beat each candidate ”head-to-head”:

Definition

(Pairwise Comparison Method)

SETUP: Single-Winner Election has k candidates

PROCESS:
(1) Voters rank all candidates
(2) Pit candidates A and B ”head-to-head”
    Count how many voters prefer A to B
    Count how many voters prefer B to A
    If A and B are tied, then each receives 1/2 point
    Else the more preferred candidate receives 1 point
    and the less preferred candidate receives 0 points
(3) Repeat Step (2) for each pair of candidates

WINNER: Candidate receiving the **most points**

Pairwise Comparison Method is typically used in Sports Drafts.

How Many Pairwise Comparisons are Necessary?

# CANDIDATES	TOTAL # PAIRWISE COMPARISONS
3	3
4	6
5	10
6	15
7	21
8	28
9	36
10	45
15	105
20	190
50	1225
100	4950

Pairwise Comparisons Method (Example)

WEX 11-1-4: Using Pairwise Comparisons Mtd, determine election winner:

	Number of Ballots				
Preference	10	7	5	5	4
1^{st} choice	A	D	B	C	B
2^{nd} choice	C	B	C	D	C
3^{rd} choice	B	A	A	A	D
4^{th} choice	D	C	D	B	A

Pairwise Comparisons Method (Example)

WEX 11-1-4: Using Pairwise Comparisons Mtd, determine election winner:

	Number of Ballots				
Preference	10	7	5	5	4
1^{st} choice	A	D	B	C	B
2^{nd} choice	C	B	C	D	C
3^{rd} choice	B	A	A	A	D
4^{th} choice	D	C	D	B	A

CANDIDATE	POINTS	$\frac{1}{2}$-POINTS
A		
B	1	
C		
D		

A versus B:

$$A = 10 + 5 = 15$$

$$B = 7 + 5 + 4 = 16 \leftarrow \text{WINNER!}$$

Pairwise Comparisons Method (Example)

WEX 11-1-4: Using Pairwise Comparisons Mtd, determine election winner:

	Number of Ballots				
Preference	10	7	5	5	4
1^{st} choice	A	D	B	C	B
2^{nd} choice	C	B	C	D	C
3^{rd} choice	B	A	A	A	D
4^{th} choice	D	C	D	B	A

CANDIDATE	POINTS	$\frac{1}{2}$-POINTS
A	1	
B	1	
C		
D		

A versus C:

$$A = 10 + 7 = 17 \leftarrow \text{WINNER!}$$

$$C = 5 + 5 + 4 = 14$$

Pairwise Comparisons Method (Example)

WEX 11-1-4: Using Pairwise Comparisons Mtd, determine election winner:

	Number of Ballots				
Preference	10	7	5	5	4
1^{st} choice	A	D	B	C	B
2^{nd} choice	C	B	C	D	C
3^{rd} choice	B	A	A	A	D
4^{th} choice	D	C	D	B	A

CANDIDATE	POINTS	$\frac{1}{2}$-POINTS
A	1	
B	1	
C		
D	1	

A versus D:

$$A = 10 + 5 = 15$$

$$D = 7 + 5 + 4 = 16 \leftarrow \text{WINNER!}$$

Pairwise Comparisons Method (Example)

WEX 11-1-4: Using Pairwise Comparisons Mtd, determine election winner:

	Number of Ballots				
Preference	10	7	5	5	4
1^{st} choice	A	D	*B*	*C*	*B*
2^{nd} choice	*C*	*B*	*C*	D	*C*
3^{rd} choice	*B*	A	A	A	D
4^{th} choice	D	*C*	D	*B*	A

CANDIDATE	POINTS	$\frac{1}{2}$-POINTS
A	1	
B	2	
C		
D	1	

B versus C:

$$B = 7 + 5 + 4 = 16 \leftarrow \text{WINNER!}$$

$$C = 10 + 5 = 15$$

Pairwise Comparisons Method (Example)

WEX 11-1-4: Using Pairwise Comparisons Mtd, determine election winner:

	Number of Ballots				
Preference	10	7	5	5	4
1^{st} choice	A	D	B	C	B
2^{nd} choice	C	B	C	D	C
3^{rd} choice	B	A	A	A	D
4^{th} choice	D	C	D	B	A

CANDIDATE	POINTS	$\frac{1}{2}$-POINTS
A	1	
B	3	
C		
D	1	

B versus D:

$$B = 10 + 5 + 4 = 19 \leftarrow \text{WINNER!}$$

$$D = 7 + 5 = 12$$

Pairwise Comparisons Method (Example)

WEX 11-1-4: Using Pairwise Comparisons Mtd, determine election winner:

	Number of Ballots				
Preference	10	7	5	5	4
1^{st} choice	A	D	B	C	B
2^{nd} choice	C	B	C	D	C
3^{rd} choice	B	A	A	A	D
4^{th} choice	D	C	D	B	A

CANDIDATE	POINTS	$\frac{1}{2}$-POINTS
A	1	
B	3	
C	1	
D	1	

C versus D:

$$C = 10 + 5 + 5 + 4 = 24 \leftarrow \text{WINNER!}$$

$$D = 7$$

Pairwise Comparisons Method (Example)

WEX 11-1-4: Using Pairwise Comparisons Mtd, determine election winner:

	Number of Ballots				
Preference	10	7	5	5	4
1^{st} choice	A	D	B	C	B
2^{nd} choice	C	B	C	D	C
3^{rd} choice	B	A	A	A	D
4^{th} choice	D	C	D	B	A

CANDIDATE	POINTS	$\frac{1}{2}$-POINTS
A	1	
B	3	
C	1	
D	1	

Since candidate B has the most points, B is the winner

Summary of the Previous Examples

	Number of Ballots				
Preference	10	7	5	5	4
1^{st} choice	A	D	B	C	B
2^{nd} choice	C	B	C	D	C
3^{rd} choice	B	A	A	A	D
4^{th} choice	D	C	D	B	A

EXAMPLE	VOTING METHOD	WINNER
WEX 11-1-1	Plurality	**A**
WEX 11-1-2	Borda Count	**C**
WEX 11-1-3	Plurality+Elimination	**D**
WEX 11-1-4	Pairwise Comparisons	**B**

As this shows, it's possible for each method to determine a different winner of the same election.

Voter Apathy & Alternative Voting Methods

Definition

(Voter Apathy)

Voter apathy is the belief that one's vote does not count.

Some voter apathy is caused by frustration with the Plurality Method.

In response to voter apathy, alternative voting methods have been proposed:

Approval Voting	Voters vote for as many candidates as they want
Instant Runoff Voting	Voters rank candidates Weakest candidate is eliminated If eliminated candidate was a voter's 1^{st} choice, then that candidate becomes voter's 2^{nd} choice (i.e vote was not wasted)

Such alternative voting methods are beyond the scope of this course.

Fin.