Defects in Voting Methods
Contemporary Math

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TTU

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Fairness in Voting Methods

With so many voting methods, which are the "best" or fairest?

It turns out there are four reasonable conditions that an election should satisfy:

- Majority Criterion
- Condorcet's Criterion
- Independence-of-Irrelevant-Alternatives (IIA) Criterion
- Monotonicity Criterion

Majority Criterion

Definition

(Majority)

A **majority** (of votes or voters) means **at least** 50% (of the votes or voters)

Definition

(Majority Criterion)

If a **majority of the voters** rank candidate X as their 1st choice,
then candidate X should win the election.

WEX 11-2-1: Given the below preferences table, explain why the Borda Count Method violates the Majority Criterion.

	Number of Ballots			
Preference	5	9	8	5
1st	A	A	C	B
2nd	C	C	B	C
3rd	B	D	D	D
4th	D	B	A	A

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The Borda Count winner (work omitted here) is C.

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However:

The total # of votes is $5 + 9 + 8 + 5 = 27$, and # of votes for A $= 5 + 9 = 14$,

$\implies$ candidate A has a majority since $14/27 \approx 0.51 > 0.50 = 50\%$

$\therefore$ The Borda Count Method violates the Majority Criterion since

the Majority candidate is A, but the Borda Count winner is C

Condorcet's Criterion

Nicholas de Condorcet (1743-1794) was a French philosopher and mathematician who believed that mathematics could be used in the social sciences as precisely as in the physical sciences.

Definition

(Condorcet Winner)

If candidate X can defeat each of the other candidates in a head-to-head vote, then candidate X is called the **Condorcet winner**.

REMARK: It's possible for an election to <u>not</u> have a Condorcet winner.

Definition

(Condorcet's Criterion)

If an election has a Condorcet winner, then the Condorcet winner should win the election.

Independence-of-Irrelevant-Alternatives (IIA) Criterion

Definition

(Independence-of-Irrelevant-Alternatives (IIA) Criterion)

If candidate X wins an election, some nonwinners are removed from the ballot, and a recount is done, then candidate X still wins the election.

Monotonicity Criterion

Definition

(Monotonicity Criterion)

If candidate X wins an election, and in a reelection all voters who change their votes only change their votes to favor X, then candidate X also wins the reelection.

DEMONSTRATION OF MONOTONICITY CRITERION VIOLATIONS ARE LENGTHY & TEDIOUS! HENCE MONOTONICITY CRITERION WILL NOT BE CONSIDERED GOING FORWARD.

Which Voting Methods can violate which Criteria??

	Can Violate Majority Criterion?	Can Violate Condorcet Criterion?	Can Violate IIA Criterion?	Can Violate Monotonicity Criterion?
Plurality	No	*YES*	*YES*	No
Borda Count	*YES*	*YES*	*YES*	No
Plurality w/ Elim.	No	*YES*	*YES*	*YES*
Pairwise Comp.	No	No	*YES*	No

This table suggests that Pairwise Comparison is the best method,
but if there are more than 5 candidates, it becomes too long & tedious to use.

Fin.