Probability: Introduction

Contemporary Math

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Random Processes & Probability Theory

Life is full of processes whose outcome cannot be predicted ahead of time:

Definition

(Random Process)

A **random process** is a process whose outcome cannot be predicted a priori.

Examples of random processes:

- Meteorology: Changes in Weather
- Finance: Changes in Stock Prices
- Gambling: Winning in Games of Chance (Blackjack, Slot Machines, ...)

If we can't predict the outcome, what's the next best thing?

Use **Probability Theory** to determine the **likelihood** of a particular outcome!

Definition

(Probability Theory)

Probability Theory is the quantitative study of uncertainty.

Basic Terminology

Definition

(Experiments, Outcomes, Sample Spaces, Events)

A **random process** is a process whose outcome cannot be predicted a priori.
An **experiment** is any observation of a random process.
The **outcomes** of an experiment are the different possible results.
The **sample space** of an experiment is the set of all possible outcomes.
An **event** is a subset of the sample space.

REMARK: The **empty set**, $\emptyset$, is the set with nothing in it.
REMARK: The **empty set**, $\emptyset$, is always a subset of the sample space.

WEX 13-1-1: Two fair coins are flipped and then their top faces are observed.
(a) Determine the sample space for the experiment.
(b) Write each event as a subset of the sample space:
Event $E_1 \equiv$ Two heads occur.
Event $E_2 \equiv$ A head & a tail occur.
Event $E_3 \equiv$ First coin is tails.

Sample Spaces, Outcomes, Events (Example)

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Let $H \equiv$ Heads, $T \equiv$ Tails. Then:

(a) Sample space $\boxed{S = \{HH, HT, TH, TT\}}$

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(a) Sample space $\boxed{S = \{HH, HT, TH, TT\}}$

(b) $\boxed{E_1 = \{HH\}}$

Sample Spaces, Outcomes, Events (Example)

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(a) Sample space $\boxed{S = \{HH, HT, TH, TT\}}$

(b) $\boxed{E_1 = \{HH\}}$ $\boxed{E_2 = \{HT, TH\}}$

Sample Spaces, Outcomes, Events (Example)

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(a) Determine the sample space for the experiment.
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Event $E_1 \equiv$ Two heads occur.
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Let $H \equiv$ Heads, $T \equiv$ Tails. Then:

(a) Sample space $\boxed{S = \{HH, HT, TH, TT\}}$

(b) $\boxed{E_1 = \{HH\}}$ $\boxed{E_2 = \{HT, TH\}}$ $\boxed{E_3 = \{TH, TT\}}$

WARNING: Order matters: HT and TH are **different outcomes**!

Probability (Definition)

Definition

(Probability)

The **probability of an outcome** in a sample space is the likelihood of the outcome, which is a number between 0 and 1 inclusive.

The **probability of an event** E, denoted $P(E)$, is the sum of the probablities of the outcomes that comprise E.

Interpretation of Probability:

$$
\begin{aligned}
\text{Probability } &= 0 & \Longrightarrow &\quad \text{Outcome/Event is impossible} \\
0 < \text{ Probability } &< \tfrac{1}{2} & \Longrightarrow &\quad \text{Outcome/Event is not likely to occur} \\
\text{Probability } &= \tfrac{1}{2} & \Longrightarrow &\quad \text{Outcome/Event has 50-50 chance of occurring} \\
\tfrac{1}{2} < \text{ Probability } &< 1 & \Longrightarrow &\quad \text{Outcome/Event is likely to occur} \\
\text{Probability } &= 1 & \Longrightarrow &\quad \text{Outcome/Event is certain to occur}
\end{aligned}
$$

Probability (Definition)

Definition

(Probability)

The **probability of an outcome** in a sample space is the likelihood of the outcome, which is a number between 0 and 1 inclusive.

The **probability of an event** E, denoted $P(E)$, is the sum of the probablities of the outcomes that comprise E.

Examples of Probability:

- "There's a 30% chance of snow tomorrow." $[P(\text{Snow tomorrow}) = 0.30]$
- "25% of adults get seven hours of sleep." $[P(\text{7 hrs of sleep}) = 0.25]$
- "All dogs play fetch." $[P(\text{Playing fetch}) = 1]$
- "There's a 1 in 1000 chance of winning." $\left[P(\text{Winning}) = \frac{1}{1000}\right]$
- "None of my cats catch mice." $[P(\text{Catch mice}) = 0]$

Measure of a Set (Definition)

Definition

(Measure of a Set)

The **measure** of a **countable set** is defined as: $m(E) = (\text{\# of elements in } E)$
The **measure** of a **1D set** is defined as: $m(E) = (\text{Length of } E)$
The **measure** of a **2D set** is defined as: $m(E) = (\text{Area of } E)$
The **measure** of a **3D set** is defined as: $m(E) = (\text{Volume of } E)$

The **measure** of the **empty set** is defined to be zero: $m(\emptyset) = 0$

Measure of a Countable Set (Example)

Definition

(Measure of a Set)

The **measure** of a **countable set** is defined as: $m(E) = (\text{\# of elements in } E)$
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The **measure** of a **3D set** is defined as: $m(E) = (\text{Volume of } E)$

The **measure** of the **empty set** is defined to be zero: $m(\emptyset) = 0$

Example: Let $S = \{\text{Heads}, \text{Tails}\}$.

Then, $m(S) = (\text{\# of elements of } S) = \boxed{2}$

Measure of a 1D Set (Example)

Definition

(Measure of a Set)

The **measure** of a **countable set** is defined as: $m(E) = (\#\text{ of elements in } E)$
The **measure** of a **1D set** is defined as: $m(E) = (\text{Length of } E)$
The **measure** of a **2D set** is defined as: $m(E) = (\text{Area of } E)$
The **measure** of a **3D set** is defined as: $m(E) = (\text{Volume of } E)$

The **measure** of the **empty set** is defined to be zero: $m(\emptyset) = 0$

Example: Let ℓ be a line segment with length 13.

Then, $m(\ell) = (\text{Length of } \ell) = \boxed{13}$

Measure of a 2D Set (Example)

Definition

(Measure of a Set)

The **measure** of a **countable set** is defined as: $m(E) = (\text{\# of elements in } E)$
The **measure** of a **1D set** is defined as: $m(E) = (\text{Length of } E)$
The **measure** of a **2D set** is defined as: $m(E) = (\text{Area of } E)$
The **measure** of a **3D set** is defined as: $m(E) = (\text{Volume of } E)$

The **measure** of the **empty set** is defined to be zero: $m(\emptyset) = 0$

<u>Example:</u> Let R be a rectangle with length 2 and width 3.

Then, $m(R) = (\text{Area of } R) = (\text{Length}) \times (\text{Width}) = 2 \times 3 = \boxed{6}$

Measure of a 3D Set (Example)

Definition

(Measure of a Set)

The **measure** of a **countable set** is defined as: $m(E) = $ (# of elements in E)
The **measure** of a **1D set** is defined as: $m(E) = $ (Length of E)
The **measure** of a **2D set** is defined as: $m(E) = $ (Area of E)
The **measure** of a **3D set** is defined as: $m(E) = $ (Volume of E)

The **measure** of the **empty set** is defined to be zero: $m(\emptyset) = 0$

Example: Let C be a cube of length 4.

Then, $m(C) = $ (Volume of C) $=$ (Length)$^3 = 4^3 = \boxed{64}$

Probability of an Event (Definition)

Definition

(Proabability of an Event)

Let S be the sample space of an experiment.
Let E be an event of the experiment.

Then the **probability** of event E occurring is defined as: $$P(E) = \frac{m(E)}{m(S)}$$

REMARK:
Often it's impractical to list every outcome of a sample space S or event E.
When computing probability, only the **measures** of E & S are needed.

Probability of an Event (Example)

WEX 13-1-2: Two fair coins are flipped and then their top sides are observed.
(a) Find the probability that two heads occur.
(b) Find the probability that the first coin is tails.

Probability of an Event (Example)

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Sample Space $S = \{HH, HT, TH, TT\}$

Probability of an Event (Example)

WEX 13-1-2: Two fair coins are flipped and then their top sides are observed.
(a) Find the probability that two heads occur.
(b) Find the probability that the first coin is tails.

Sample Space $S = \{HH, HT, TH, TT\}$

(a) Let $E_1 \equiv$ Two heads occur $= \{HH\}$.

Probability of an Event (Example)

WEX 13-1-2: Two fair coins are flipped and then their top sides are observed.
(a) Find the probability that two heads occur.
(b) Find the probability that the first coin is tails.

Sample Space $S = \{HH, HT, TH, TT\}$

(a) Let $E_1 \equiv$ Two heads occur $= \{HH\}$.

Then $P(E_1) = \dfrac{m(E_1)}{m(S)} = \boxed{\dfrac{1}{4}}$

Probability of an Event (Example)

WEX 13-1-2: Two fair coins are flipped and then their top sides are observed.
(a) Find the probability that two heads occur.
(b) Find the probability that the first coin is tails.

Sample Space $S = \{HH, HT, TH, TT\}$

(a) Let $E_1 \equiv$ Two heads occur $= \{HH\}$.

Then $P(E_1) = \dfrac{m(E_1)}{m(S)} = \boxed{\dfrac{1}{4}}$

(b) Let $E_2 \equiv$ First coin is tails $= \{TH, TT\}$.

Probability of an Event (Example)

WEX 13-1-2: Two fair coins are flipped and then their top sides are observed.
(a) Find the probability that two heads occur.
(b) Find the probability that the first coin is tails.

Sample Space $S = \{HH, HT, TH, TT\}$

(a) Let $E_1 \equiv$ Two heads occur $= \{HH\}$.

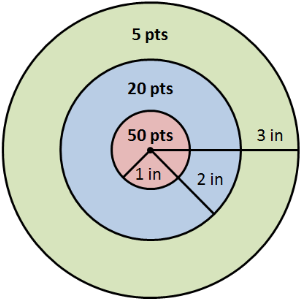
Then $P(E_1) = \dfrac{m(E_1)}{m(S)} = \boxed{\dfrac{1}{4}}$

(b) Let $E_2 \equiv$ First coin is tails $= \{TH, TT\}$.

Then $P(E_2) = \dfrac{m(E_2)}{m(S)} = \dfrac{2}{4} = \boxed{\dfrac{1}{2}}$

Probability of an Event (Example)

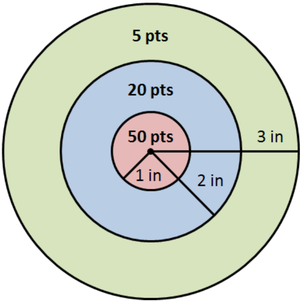
WEX 13-1-3: You throw a dart at the following target:



(a) What is the probability that you earn 5 points?
(b) What is the probability that you earn 20 points?
(c) What is the probability that you earn 50 points?

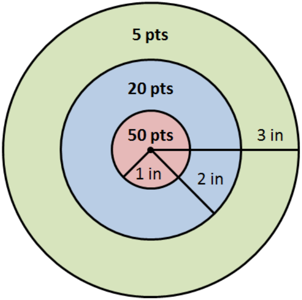
Probability of an Event (Example)

<u>WEX 13-1-3:</u> You throw a dart at the following target:



Sample Space $S = (\text{Entire Target})$

WEX 13-1-3: You throw a dart at the following target:

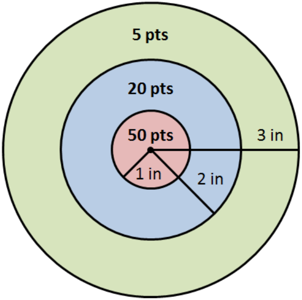


Sample Space $S = $ (Entire Target)

$\implies m(S) = $ (Area of Circle with radius$3) = \pi(3)^2 = 9\pi$

Probability of an Event (Example)

WEX 13-1-3: You throw a dart at the following target:



Sample Space $S =$ (Entire Target)

$\implies m(S) =$ (Area of Circle with radius3) $= \pi(3)^2 = 9\pi$

(a) Let event $E_1 \equiv$ "Earn 5pts"

$$P(E_1) = \frac{m(E_1)}{m(S)} = \frac{\text{(Area of Green)}}{m(S)} = \frac{\pi(3)^2 - \pi(2)^2}{9\pi} = \frac{9\pi - 4\pi}{9\pi} = \frac{5\pi}{9\pi} = \boxed{\frac{5}{9}}$$

Probability of an Event (Example)

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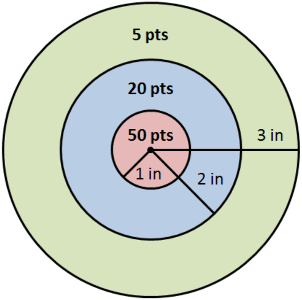


Sample Space $S = $ (Entire Target)

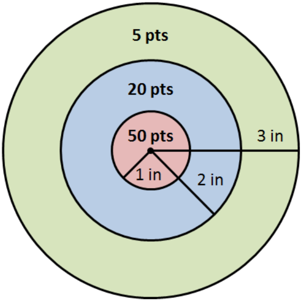
$\implies m(S) = $ (Area of Circle with radius$3) = \pi(3)^2 = 9\pi$

(b) Let event $E_2 \equiv$ "Earn 20pts"

$$P(E_2) = \frac{m(E_2)}{m(S)} = \frac{\text{(Area of Blue)}}{m(S)} = \frac{\pi(2)^2 - \pi(1)^2}{9\pi} = \frac{4\pi - \pi}{9\pi} = \frac{3\pi}{9\pi} = \boxed{\frac{1}{3}}$$

Probability of an Event (Example)

WEX 13-1-3: You throw a dart at the following target:



Sample Space $S =$ (Entire Target)

$\implies m(S) =$ (Area of Circle with radius3) $= \pi(3)^2 = 9\pi$

(c) Let event $E_3 \equiv$ "Earn 50pts"

$$P(E_3) = \frac{m(E_3)}{m(S)} = \frac{\text{(Area of Red)}}{m(S)} = \frac{\pi(1)^2}{9\pi} = \frac{\pi}{9\pi} = \boxed{\frac{1}{9}}$$

Probability of an Event (Example)

WEX 13-1-4: Given the following table of a study conducted about marital status:

	Single	Married	Divorced	Widowed
Male	123	330	45	10
Female	151	370	32	18

(a) What is the probability that a randomly chosen person is a single man?
(b) What is the probability that a randomly chosen person is married?
(c) What is the probability that a randomly chosen divorced person is male?
(d) What is the probability that a randomly chosen female is widowed?

Probability of an Event (Example)

WEX 13-1-4: Given the following table of a study conducted about marital status:

	Single	Married	Divorced	Widowed
Male	123	330	45	10
Female	151	370	32	19

(a) What is the probability that a randomly chosen person is a single man?

Probability of an Event (Example)

WEX 13-1-4: Given the following table of a study conducted about marital status:

	Single	Married	Divorced	Widowed
Male	123	330	45	10
Female	151	370	32	19

(a) What is the probability that a randomly chosen person is a single man?

(a) Sample Space $S_1 \equiv$ (All people in study)
$\implies m(S_1) = 123 + 151 + 330 + 370 + 45 + 32 + 10 + 19 = 1080$

WEX 13-1-4: Given the following table of a study conducted about marital status:

	Single	Married	Divorced	Widowed
Male	123	330	45	10
Female	151	370	32	19

(a) What is the probability that a randomly chosen person is a single man?

(a) Sample Space $S_1 \equiv$ (All people in study)

$\implies m(S_1) = 123 + 151 + 330 + 370 + 45 + 32 + 10 + 19 = 1080$

Event $E_1 \equiv$ (Single Man) $\implies m(E_1) = 123$

Probability of an Event (Example)

WEX 13-1-4: Given the following table of a study conducted about marital status:

	Single	Married	Divorced	Widowed
Male	123	330	45	10
Female	151	370	32	19

(a) What is the probability that a randomly chosen person is a single man?

(a) Sample Space $S_1 \equiv$ (All people in study)

$$\implies m(S_1) = 123 + 151 + 330 + 370 + 45 + 32 + 10 + 19 = 1080$$

Event $E_1 \equiv$ (Single Man) $\implies m(E_1) = 123$

$$\implies P(E_1) = \frac{m(E_1)}{m(S_1)} = \frac{123}{1080} = \boxed{\frac{41}{360}} \approx 0.1138 = 11.38\% \text{ chance}$$

Probability of an Event (Example)

WEX 13-1-4: Given the following table of a study conducted about marital status:

	Single	Married	Divorced	Widowed
Male	123	330	45	10
Female	151	370	32	19

(b) What is the probability that a randomly chosen person is married?

WEX 13-1-4: Given the following table of a study conducted about marital status:

	Single	Married	Divorced	Widowed
Male	123	330	45	10
Female	151	370	32	19

(b) What is the probability that a randomly chosen person is married?

(b) Sample Space $S_2 \equiv$ (All people in study)

$\implies m(S_2) = 123 + 151 + 330 + 370 + 45 + 32 + 10 + 19 = 1080$

WEX 13-1-4: Given the following table of a study conducted about marital status:

	Single	Married	Divorced	Widowed
Male	123	330	45	10
Female	151	370	32	19

(b) What is the probability that a randomly chosen person is married?

(b) Sample Space $S_2 \equiv$ (All people in study)
$\implies m(S_2) = 123 + 151 + 330 + 370 + 45 + 32 + 10 + 19 = 1080$

Event $E_2 \equiv$ (Married Person) $\implies m(E_2) = 330 + 370 = 700$

Probability of an Event (Example)

WEX 13-1-4: Given the following table of a study conducted about marital status:

	Single	Married	Divorced	Widowed
Male	123	330	45	10
Female	151	370	32	19

(b) What is the probability that a randomly chosen person is married?

(b) Sample Space $S_2 \equiv$ (All people in study)

$\implies m(S_2) = 123 + 151 + 330 + 370 + 45 + 32 + 10 + 19 = 1080$

Event $E_2 \equiv$ (Married Person) $\implies m(E_2) = 330 + 370 = 700$

$$\implies P(E_2) = \frac{m(E_2)}{m(S_2)} = \frac{700}{1080} = \boxed{\frac{35}{54}} \approx 0.6481 = 64.81\% \text{ chance}$$

Probability of an Event (Example)

WEX 13-1-4: Given the following table of a study conducted about marital status:

	Single	Married	Divorced	Widowed
Male	123	330	45	10
Female	151	370	32	19

(c) What is the probability that a randomly chosen divorced person is male?

Probability of an Event (Example)

WEX 13-1-4: Given the following table of a study conducted about marital status:

	Single	Married	Divorced	Widowed
Male	123	330	45	10
Female	151	370	32	19

(c) What is the probability that a randomly chosen divorced person is male?

(c) Sample Space $S_3 \equiv$ (All Divorced People)
$\implies m(S_3) = 45 + 32 = 77$

WEX 13-1-4: Given the following table of a study conducted about marital status:

	Single	Married	Divorced	Widowed
Male	123	330	45	10
Female	151	370	32	19

(c) What is the probability that a randomly chosen divorced person is male?

(c) Sample Space $S_3 \equiv$ (All Divorced People)
$\implies m(S_3) = 45 + 32 = 77$

Event $E_3 \equiv$ (Divorced Man) $\implies m(E_3) = 45$

Probability of an Event (Example)

WEX 13-1-4: Given the following table of a study conducted about marital status:

	Single	Married	Divorced	Widowed
Male	123	330	45	10
Female	151	370	32	19

(c) What is the probability that a randomly chosen divorced person is male?

(c) Sample Space $S_3 \equiv$ (All Divorced People)

$\implies m(S_3) = 45 + 32 = 77$

Event $E_3 \equiv$ (Divorced Man) $\implies m(E_3) = 45$

$$\implies P(E_3) = \frac{m(E_3)}{m(S_3)} = \boxed{\frac{45}{77}} \approx 0.5844 = 58.44\% \text{ chance}$$

WEX 13-1-4: Given the following table of a study conducted about marital status:

	Single	Married	Divorced	Widowed
Male	123	330	45	10
Female	151	370	32	19

(d) What is the probability that a randomly chosen female is widowed?

Probability of an Event (Example)

WEX 13-1-4: Given the following table of a study conducted about marital status:

	Single	Married	Divorced	Widowed
Male	123	330	45	10
Female	151	370	32	19

(d) What is the probability that a randomly chosen female is widowed?

(d) Sample Space $S_4 \equiv$ (All Women)
$\implies m(S_4) = 151 + 370 + 32 + 19 = 572$

Probability of an Event (Example)

WEX 13-1-4: Given the following table of a study conducted about marital status:

	Single	Married	Divorced	Widowed
Male	123	330	45	10
Female	151	370	32	19

(d) What is the probability that a randomly chosen female is widowed?

(d) Sample Space $S_4 \equiv$ (All Women)
$\implies m(S_4) = 151 + 370 + 32 + 19 = 572$

Event $E_4 \equiv$ (Widowed Woman) $\implies m(E_4) = 19$

Probability of an Event (Example)

WEX 13-1-4: Given the following table of a study conducted about marital status:

	Single	Married	Divorced	Widowed
Male	123	330	45	10
Female	151	370	32	19

(d) What is the probability that a randomly chosen female is widowed?

(d) Sample Space $S_4 \equiv$ (All Women)

$\implies m(S_4) = 151 + 370 + 32 + 19 = 572$

Event $E_4 \equiv$ (Widowed Woman) $\implies m(E_4) = 19$

$$\implies P(E_4) = \frac{m(E_4)}{m(S_4)} = \boxed{\frac{19}{572}} \approx 0.0332 = 3.32\% \text{ chance}$$

Properties of Probability

Proposition

(Properties of Probability)

Let S be a sample space and E be an event. Then:

(a) $0 \leq P(E) \leq 1$
(b) $P(\emptyset) = 0$
(c) $P(S) = 1$

Properties of Probability

Proposition

(Properties of Probability)

Let S *be a sample space and* E *be an event.* *Then:*

(a) $0 \leq P(E) \leq 1$
(b) $P(\emptyset) = 0$
(c) $P(S) = 1$

PROOF:

(a) $0 \leq m(E) \leq m(S) \implies \dfrac{0}{m(S)} \leq \dfrac{m(E)}{m(S)} \leq \dfrac{m(S)}{m(S)} \implies 0 \leq P(E) \leq 1$

Properties of Probability

Proposition

(Properties of Probability)

Let S *be a sample space and* E *be an event. Then:*

(a) $0 \leq P(E) \leq 1$
(b) $P(\emptyset) = 0$
(c) $P(S) = 1$

PROOF:

(b) $P(\emptyset) = \dfrac{m(\emptyset)}{m(S)} = \dfrac{0}{m(S)} = 0$

Properties of Probability

Proposition

(Properties of Probability)

Let S be a sample space and E be an event. Then:

(a) $0 \leq P(E) \leq 1$
(b) $P(\emptyset) = 0$
(c) $P(S) = 1$

PROOF:

(c) $$P(S) = \frac{m(S)}{m(S)} = 1$$

QED

Complement of an Event (Definition)

Definition

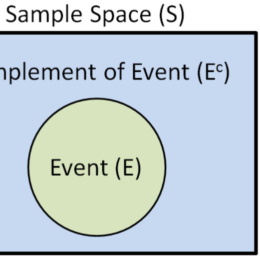
(Complement of an Event)

Let S be a sample space and E be an event.

Then the **complement** of event E, denoted E^c, is the set of all outcomes in S that are not in E.

REMARK: The complement of the sample space is the empty set: $S^c = \emptyset$
REMARK: The complement of the empty set is the sample space: $\emptyset^c = S$

REMARK: The textbook denotes the complement of E as E'.



Complement of an Event (Example)

Example:

Let sample space $S = \{HH, HT, TH, TT\}$
and events $E_1 = \{HH\}$, $E_2 = \{HH, HT\}$, and $E_3 = \{HH, HT, TH, TT\}$.

Then $E_1^c = \{HT, TH, TT\}$, $E_2^c = \{TH, TT\}$, and $E_3^c = \emptyset$

$$\implies P\left(E_1^c\right) = \frac{m\left(E_1^c\right)}{m(S)} = \frac{3}{4}$$

$$\implies P\left(E_2^c\right) = \frac{m\left(E_2^c\right)}{m(S)} = \frac{2}{4} = \frac{1}{2}$$

$$\implies P\left(E_3^c\right) = \frac{m\left(E_3^c\right)}{m(S)} = \frac{0}{4} = 0$$

Odds (Definition)

Definition

(Odds in Favor of an Event)

$$(\textbf{Odds in favor} \text{ of event } E) = \frac{P(E)}{P\left(E^c\right)}$$

Definition

(Odds Against an Event)

$$(\textbf{Odds against} \text{ an event } E) = \frac{P\left(E^c\right)}{P\left(E\right)}$$

Odds (Example)

WEX 13-1-5: Two coins are flipped and then their top sides are observed.
(a) What are the odds in favor of two heads occurring?
(b) What are the odds against two heads occurring?

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Let $E \equiv$ (Two heads occur) $= \{HH\}$.

Then $E^c \equiv$ (Two heads do not occur) $= \{HT, TH, TT\}$

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$$\implies P(E) = \frac{m(E)}{m(S)} = \frac{1}{4} \quad \text{and} \quad P(E^c) = \frac{m(E^c)}{m(S)} = \frac{3}{4}$$

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(a) (Odds in favor of two heads occurring) $=$ (Odds in favor of E)

$$= \frac{P(E)}{P(E^c)} = \frac{1/4}{3/4} = \frac{1}{4} \times \frac{4}{3} = \frac{1}{3} = \boxed{\text{1 to 3}}$$

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Let $E \equiv$ (Two heads occur) $= \{HH\}$.

Then $E^c \equiv$ (Two heads do not occur) $= \{HT, TH, TT\}$

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(b) (Odds against two heads occurring) $=$ (Odds against E)

$$= \frac{P(E^c)}{P(E)} = \frac{3/4}{1/4} = \frac{3}{4} \times \frac{4}{1} = \frac{3}{1} = \boxed{\text{3 to 1}}$$

Fin.