Probability: Expected Value

Contemporary Math

Josh Engwer

TTU

30 July 2015

Expected Value (Definition)

How can one use probabilities to determine long-term expectations?

Definition

(Expected Value)

Suppose an experiment has a sample space with N possible outcomes with probabilities $P_1, P_2, \ldots, P_N$.
Moreover, assume each outcome has an associated value with it that are labeled $V_1, V_2, \ldots, V_N$.

Then, the **expected value** of the experiment is:

$$EV = \sum_{k=1}^{N} P_k V_k = P_1 V_1 + P_2 V_2 + \cdots + P_N V_N$$

Expected value is particularly used in the following situations:

- How much money is expected to be gained/lost when playing a game of chance repeatedly?
- How much should an insurance policy premium be?
- How much profit is expected to be gained/lost long-term?

Expected Value (Example)

EX 13-4-1: Given the following table of the probabilities & values associated with the four outcomes of an experiment:

OUTCOME	**PROBABILITY**	**VALUE**
A	0.37	-5
B	0.20	6
C	0.43	-3

Compute the expected value for the experiment.

Expected Value (Example)

EX 13-4-1: Given the following table of the probabilities & values associated with the four outcomes of an experiment:

OUTCOME	PROBABILITY	VALUE
A	0.37	-5
B	0.20	6
C	0.43	-3

Compute the expected value for the experiment.

$$\text{(Expected Value)} = P_A V_A + P_B V_B + P_C V_C$$

Expected Value (Example)

EX 13-4-1: Given the following table of the probabilities & values associated with the four outcomes of an experiment:

OUTCOME	PROBABILITY	VALUE
A	0.37	-5
B	0.20	6
C	0.43	-3

Compute the expected value for the experiment.

$$\begin{aligned}\text{(Expected Value)} &= P_A V_A + P_B V_B + P_C V_C \\ &= (0.37)(-5) + (0.20)(6) + (0.43)(-3)\end{aligned}$$

Expected Value (Example)

EX 13-4-1: Given the following table of the probabilities & values associated with the four outcomes of an experiment:

OUTCOME	PROBABILITY	VALUE
A	0.37	-5
B	0.20	6
C	0.43	-3

Compute the expected value for the experiment.

$$\begin{aligned}
\text{(Expected Value)} &= P_A V_A + P_B V_B + P_C V_C \\
&= (0.37)(-5) + (0.20)(6) + (0.43)(-3) \\
&= \boxed{-1.94}
\end{aligned}$$

Games of Chance (Expected Value)

Proposition

(Expected Value of Games of Chance)

*If the experiment is playing a **game of chance**, then:*

- *The game is **fair** if the game has an **expected value** of **zero**:* $EV = 0$
- *The game is **unfair** if it has an **expected value** that's **not zero**:* $EV \neq 0$

REMARK: Casinos ensure that their games have a **negative expected value** so that they make money off their customers.

Games of Chance (Example)

WEX 13-4-2: You pay \$1.00 to play roulette. A roulette wheel has 38 slots.
If the ball lands on the slot labeled 20, you win \$30.
Otherwise, you lose the dollar you paid to play the game.

(a) Find the expected value for playing a game of roulette.
(b) If you play 1000 consecutive games of roulette, what should you expect?

Games of Chance (Example)

<u>WEX 13-4-2:</u> You pay $1.00 to play roulette. A roulette wheel has 38 slots. If the ball lands on the slot labeled 20, you win $30.
Otherwise, you lose the dollar you paid to play the game.

(a) Find the expected value for playing a game of roulette.
(b) If you play 1000 consecutive games of roulette, what should you expect?

Sample Space $S = \{\text{slot "}1\text{"}, \text{slot "}2\text{"}, \cdots, \text{slot "}37\text{"}, \text{slot "}38\text{"}\}$

Event $E \equiv (\text{Ball lands on slot "20"}) = \{\text{slot "}20\text{"}\}$

Games of Chance (Example)

WEX 13-4-2: You pay \$1.00 to play roulette. A roulette wheel has 38 slots.
If the ball lands on the slot labeled 20, you win \$30.
Otherwise, you lose the dollar you paid to play the game.

(a) Find the expected value for playing a game of roulette.
(b) If you play 1000 consecutive games of roulette, what should you expect?

Sample Space $S = \{\text{slot "1"}, \text{slot "2"}, \cdots, \text{slot "37"}, \text{slot "38"}\}$

Event $E \equiv$ (Ball lands on slot "20") $= \{\text{slot "20"}\}$

$$P(\text{Ball lands on slot "20"}) = P(E) = \frac{m(E)}{m(S)} = \frac{1}{38}$$

$$P(\text{Ball does not land on slot "20"}) = P(E^c) = 1 - P(E) = 1 - \frac{1}{38} = \frac{37}{38}$$

Games of Chance (Example)

<u>WEX 13-4-2:</u> You pay \$1.00 to play roulette. A roulette wheel has 38 slots. If the ball lands on the slot labeled 20, you win \$30.
Otherwise, you lose the dollar you paid to play the game.

(a) Find the expected value for playing a game of roulette.
(b) If you play 1000 consecutive games of roulette, what should you expect?

Sample Space $S = \{\text{slot }"1", \text{slot }"2", \cdots, \text{slot }"37", \text{slot }"38"\}$

Event $E \equiv$ (Ball lands on slot "20") $= \{\text{slot }"20"\}$

$$P(\text{Ball lands on slot "20"}) = P(E) = \frac{m(E)}{m(S)} = \frac{1}{38}$$

$$P(\text{Ball does not land on slot "20"}) = P(E^c) = 1 - P(E) = 1 - \frac{1}{38} = \frac{37}{38}$$

EVENT	PROBABILITY	GAIN/LOSS
E	$\frac{1}{38}$	\$30
E^c	$\frac{37}{38}$	$-\$1$

WEX 13-4-2: You pay \$1.00 to play roulette. A roulette wheel has 38 slots. If the ball lands on the slot labeled 20, you win \$30. Otherwise, you lose the dollar you paid to play the game.

(a) Find the expected value for playing a game of roulette.
(b) If you play 1000 consecutive games of roulette, what should you expect?

EVENT	PROBABILITY	GAIN/LOSS
E	$\frac{1}{38}$	$\$30$
E^c	$\frac{37}{38}$	$-\$1$

(a) (Expected Value) $= (\$30)P(E) + (-\$1)P(E^c)$

WEX 13-4-2: You pay \$1.00 to play roulette. A roulette wheel has 38 slots. If the ball lands on the slot labeled 20, you win \$30.
Otherwise, you lose the dollar you paid to play the game.

(a) Find the expected value for playing a game of roulette.
(b) If you play 1000 consecutive games of roulette, what should you expect?

EVENT	PROBABILITY	GAIN/LOSS
E	$\frac{1}{38}$	$\$30$
E^c	$\frac{37}{38}$	$-\$1$

$$\begin{aligned} \text{(a) (Expected Value)} &= (\$30)P(E) + (-\$1)P(E^c) \\ &= (\$30)\left(\tfrac{1}{38}\right) + (-\$1)\left(\tfrac{37}{38}\right) \end{aligned}$$

Games of Chance (Example)

WEX 13-4-2: You pay $1.00 to play roulette. A roulette wheel has 38 slots.
If the ball lands on the slot labeled 20, you win $30.
Otherwise, you lose the dollar you paid to play the game.

(a) Find the expected value for playing a game of roulette.
(b) If you play 1000 consecutive games of roulette, what should you expect?

EVENT	PROBABILITY	GAIN/LOSS
E	$\frac{1}{38}$	$\$30$
E^c	$\frac{37}{38}$	$-\$1$

(a) (Expected Value)
$$\begin{aligned} &= (\$30)P(E) + (-\$1)P(E^c) \\ &= (\$30)\left(\frac{1}{38}\right) + (-\$1)\left(\frac{37}{38}\right) \\ &= -\frac{7}{38} \end{aligned}$$

Games of Chance (Example)

WEX 13-4-2: You pay \$1.00 to play roulette. A roulette wheel has 38 slots. If the ball lands on the slot labeled 20, you win \$30.
Otherwise, you lose the dollar you paid to play the game.

(a) Find the expected value for playing a game of roulette.
(b) If you play 1000 consecutive games of roulette, what should you expect?

EVENT	PROBABILITY	GAIN/LOSS
E	$\frac{1}{38}$	$\$30$
E^c	$\frac{37}{38}$	$-\$1$

(a) (Expected Value)
$$\begin{aligned} &= (\$30)P(E) + (-\$1)P(E^c) \\ &= (\$30)\left(\tfrac{1}{38}\right) + (-\$1)\left(\tfrac{37}{38}\right) \\ &= -\tfrac{7}{38} \approx \boxed{-\$0.18} \end{aligned}$$

Games of Chance (Example)

WEX 13-4-2: You pay \$1.00 to play roulette. A roulette wheel has 38 slots. If the ball lands on the slot labeled 20, you win \$30. Otherwise, you lose the dollar you paid to play the game.

(a) Find the expected value for playing a game of roulette.
(b) If you play 1000 consecutive games of roulette, what should you expect?

EVENT	PROBABILITY	GAIN/LOSS
E	$\frac{1}{38}$	$\$30$
E^c	$\frac{37}{38}$	$-\$1$

(a) (Expected Value)

$$\begin{aligned} &= (\$30)P(E) + (-\$1)P(E^c) \\ &= (\$30)\left(\tfrac{1}{38}\right) + (-\$1)\left(\tfrac{37}{38}\right) \\ &= -\tfrac{7}{38} \approx \boxed{-\$0.18} \end{aligned}$$

(b) (1000 games)($-\$0.18$ per game) = $-\$180 \implies$ Expect to lose \$180

Fin

Fin.