

# Statistics: Mean, Median, Mode

## Contemporary Math

Josh Engwer

TTU

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# Summarizing Data by Measuring its Central Tendency

One way to summarize data is to **visualize** the data using:

- Bar Graphs
- Histograms
- Stem-and-Leaf Displays

A way to **numerically** summarize data is to measure its **central tendency**:

- Mean
- Median
- Mode

Summarizing data comprises **descriptive statistics**.

# Mean of a Data Set (Definition)

## Definition

(Mean of a Data Set)

Given a data set with  $n$  data values.

Then, the **mean** is the average of the data set.

$$\bar{x} = \frac{\sum x}{n}$$

NOTATION:  $\sum x$  means the **sum** of all the data values in the data set.

# Median of a Data Set (Definition)

## Definition

(Median of a Data Set)

Given a data set with  $n$  data values.

Then the **median** is the middle value in the sorted data set.

If  $n$  is **odd**, then the median is the value in the **middle position**.

If  $n$  is **even**, then the median is the **average** of the **two middle values**.

# Mode of a Data Set (Definition)

## Definition

(Mode of a Data Set)

Given a data set with  $n$  data values.

Then the **mode** is the data value that occurs most frequently.

If two values occur most frequently, then each is a mode.

If more than two values occur most frequently, then there is **no mode**.

# Mean, Median, Mode of a Data Set (Example)

**WEX 14-2-1**: Given the following data set:

3, 15, 8, 11, 15

- (a) Compute the mean of the data set.
- (b) Compute the median of the data set.
- (c) Compute the mode of the data set.

# Mean, Median, Mode of a Data Set (Example)

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- (a) There are 5 data values  $\implies n = 5$

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(a) There are 5 data values  $\implies n = 5$

$$\bar{x} = \frac{\sum x}{n}$$



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(a) There are 5 data values  $\implies n = 5$

$$\bar{x} = \frac{\sum x}{n} = \frac{3 + 15 + 8 + 11 + 15}{5}$$

# Mean, Median, Mode of a Data Set (Example)

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- (a) Compute the mean of the data set.
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(a) There are 5 data values  $\implies n = 5$

$$\bar{x} = \frac{\sum x}{n} = \frac{3 + 15 + 8 + 11 + 15}{5} = \frac{52}{5} = \boxed{10.4}$$

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(b) Sort data: 3, 8, 11, 15, 15

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(b) Sort data: 3, 8, 11, 15, 15

Since  $n$  is **odd**, the sorted data set has **one middle value**.

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(b) Sort data: 3, 8, **11**, 15, 15

Since  $n$  is **odd**, the sorted data set has **one middle value**.

$\implies$  Median = (Middle Value of sorted data set (in **blue**)) =  $\boxed{11}$

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(b) Sort data: 3, 8, 11, 15, 15

Since  $n$  is **odd**, the sorted data set has **one middle value**.

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(a) There are 5 data values  $\implies n = 5$

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(b) Sort data: 3, 8, 11, 15, 15

Since  $n$  is **odd**, the sorted data set has **one middle value**.

$$\implies \text{Median} = (\text{Middle Value of sorted data set}) = \boxed{11}$$

(c) The most frequent value is 15 (in **blue**).

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Since  $n$  is **odd**, the sorted data set has one middle value.

$$\implies \text{Median} = (\text{Middle Value of sorted data set}) = \boxed{11}$$

(c) The most frequent value is 15 (in **blue**).

$$\implies \text{Mode} = (\text{Most Frequent Value}) = \boxed{15}$$



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(b) Sort data: 3, 8, 11, 15, 15

Since  $n$  is **odd**, the sorted data set has **one middle value**.

$$\implies \text{Median} = (\text{Middle Value of sorted data set}) = \boxed{11}$$

(c) The most frequent value is 15.

$$\implies \text{Mode} = (\text{Most Frequent Value}) = \boxed{15}$$

# Mean, Median, Mode of a Data Set (Example)

**WEX 14-2-2:** Given the following data set:

11, 3, 15, 8, 11, 15

- (a) Compute the mean of the data set.
- (b) Compute the median of the data set.
- (c) Compute the mode of the data set.

# Mean, Median, Mode of a Data Set (Example)

**WEX 14-2-2:** Given the following data set:

11, 3, 15, 8, 11, 15

- (a) Compute the mean of the data set.
  - (b) Compute the median of the data set.
  - (c) Compute the mode of the data set.
- (a) There are 6 data values  $\implies n = 6$

# Mean, Median, Mode of a Data Set (Example)

**WEX 14-2-2:** Given the following data set:

11, 3, 15, 8, 11, 15

- (a) Compute the mean of the data set.
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- (c) Compute the mode of the data set.

(a) There are 6 data values  $\implies n = 6$

$$\bar{x} = \frac{\sum x}{n}$$

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- (a) Compute the mean of the data set.
- (b) Compute the median of the data set.
- (c) Compute the mode of the data set.

(a) There are 6 data values  $\implies n = 6$

$$\bar{x} = \frac{\sum x}{n} = \frac{11 + 3 + 15 + 8 + 11 + 15}{6}$$

# Mean, Median, Mode of a Data Set (Example)

**WEX 14-2-2:** Given the following data set:

11, 3, 15, 8, 11, 15

- (a) Compute the mean of the data set.
- (b) Compute the median of the data set.
- (c) Compute the mode of the data set.

(a) There are 6 data values  $\implies n = 6$

$$\bar{x} = \frac{\sum x}{n} = \frac{11 + 3 + 15 + 8 + 11 + 15}{6} = \frac{63}{6} = \boxed{10.5}$$

# Mean, Median, Mode of a Data Set (Example)

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$$\bar{x} = \frac{\sum x}{n} = \frac{11 + 3 + 15 + 8 + 11 + 15}{6} = \frac{63}{6} = \boxed{10.5}$$

(b) Sort data: 3, 8, 11, 11, 15, 15

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$$\bar{x} = \frac{\sum x}{n} = \frac{11 + 3 + 15 + 8 + 11 + 15}{6} = \frac{63}{6} = \boxed{10.5}$$

(b) Sort data: 3, 8, 11, 11, 15, 15

Since  $n$  is **even**, the sorted data set has **two middle values**.



# Mean, Median, Mode of a Data Set (Example)

**WEX 14-2-2:** Given the following data set:

11, 3, 15, 8, 11, 15

- (a) Compute the mean of the data set.
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(a) There are 6 data values  $\implies n = 6$

$$\bar{x} = \frac{\sum x}{n} = \frac{11 + 3 + 15 + 8 + 11 + 15}{6} = \frac{63}{6} = \boxed{10.5}$$

(b) Sort data: 3, 8, 11, 11, 15, 15

Since  $n$  is **even**, the sorted data set has **two middle values**.

$$\implies \text{Median} = (\text{Average of two Middle Values (in blue)}) = \frac{11 + 11}{2} = \boxed{11}$$

# Mean, Median, Mode of a Data Set (Example)

**WEX 14-2-2:** Given the following data set:

11, 3, 15, 8, 11, 15

- (a) Compute the mean of the data set.
- (b) Compute the median of the data set.
- (c) Compute the mode of the data set.

(a) There are 6 data values  $\implies n = 6$

$$\bar{x} = \frac{\sum x}{n} = \frac{11 + 3 + 15 + 8 + 11 + 15}{6} = \frac{63}{6} = \boxed{10.5}$$

(b) Sort data: 3, 8, 11, 11, 15, 15

Since  $n$  is **even**, the sorted data set has **two middle values**.

$$\implies \text{Median} = (\text{Average of two Middle Values}) = \frac{11 + 11}{2} = \frac{22}{2} = \boxed{11}$$

# Mean, Median, Mode of a Data Set (Example)

**WEX 14-2-2:** Given the following data set:

11, 3, 15, 8, 11, 15

- (a) Compute the mean of the data set.
- (b) Compute the median of the data set.
- (c) Compute the mode of the data set.

(a) There are 6 data values  $\implies n = 6$

$$\bar{x} = \frac{\sum x}{n} = \frac{11 + 3 + 15 + 8 + 11 + 15}{6} = \frac{63}{6} = \boxed{10.5}$$

(b) Sort data: 3, 8, 11, 11, 15, 15

Since  $n$  is **even**, the sorted data set has **two middle values**.

$$\implies \text{Median} = (\text{Average of two Middle Values}) = \frac{11 + 11}{2} = \frac{22}{2} = \boxed{11}$$

(c) The most frequent values are 11 & 15 (in **blue**).

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$$\bar{x} = \frac{\sum x}{n} = \frac{11 + 3 + 15 + 8 + 11 + 15}{6} = \frac{63}{6} = \boxed{10.5}$$

(b) Sort data: 3, 8, 11, 11, 15, 15

Since  $n$  is **even**, the sorted data set has **two middle values**.

$$\implies \text{Median} = (\text{Average of two Middle Values}) = \frac{11 + 11}{2} = \frac{22}{2} = \boxed{11}$$

(c) The most frequent values are 11 & 15.

$$\implies \text{Mode} = (\text{Two Most Frequent Values}) = \boxed{11, 15}$$

# Frequency Distributions (Definitions)

A **frequency distribution** is a data set represented by a frequency table.

For example, the data set

2, 7, 2, 12, 12, 2, 7, 2, 7, 12, 2, 12, 12, 2

is equivalent to the frequency distribution

<b>DATA VALUE</b> ( $x$ )	<b>FREQUENCY</b> ( $f$ )
2	6
7	3
12	5

# Mean of a Frequency Distribution (Definition)

## Definition

(Mean of a Frequency Distribution)

Given a data set with  $n$  data values.

Then, the **mean** is the average of the data set.

$$\bar{x} = \frac{\sum(x \cdot f)}{\sum f}$$

### NOTATION:

$\sum(x \cdot f)$  means the **sum** of the products of each data value with its frequency.

$\sum f$  means the **sum** of all the frequencies.

# Median of a Frequency Distribution (Definition)

## Definition

(Median of a Frequency Distribution)

Given a data set with  $n$  data values.

The **median** is the middle value in the sorted frequency distribution.

If  $\sum f$  is **odd**, then the median is the value in the  $\left[ \frac{\sum f}{2} \right]$ -th position.

If  $\sum f$  is **even**, then the median is the **average** of the values  
in the  $\left( \frac{\sum f}{2} \right)$ -th &  $\left[ \left( \frac{\sum f}{2} \right) + 1 \right]$ -st positions.

# Mode of a Frequency Distribution (Definition)

## Definition

(Mode of a Frequency Distribution)

Given a data set with  $n$  data values.

The **mode** is the data value with the highest frequency.

If two values occur most frequently, then each is a mode.

If more than two values occur most frequently, then there is **no mode**.



# Mean, Median, Mode of a Freq. Dist. (Example)

**WEX 14-2-3:** Given the following frequency distribution:

<b>DATA VALUE</b>	<b>FREQUENCY</b>
$(x)$	$(f)$
6	23
9	12
20	31

- Compute the mean of the frequency distribution.
- Compute the median of the frequency distribution.
- Compute the mode of the frequency distribution.

# Mean, Median, Mode of a Freq. Dist. (Example)

**WEX 14-2-3:** Given the following frequency distribution:

DATA VALUE ( $x$ )	FREQUENCY ( $f$ )
6	23
9	12
20	31

- (a) Compute the mean of the frequency distribution.
- (b) Compute the median of the frequency distribution.
- (c) Compute the mode of the frequency distribution.

(a) 
$$\bar{x} = \frac{\sum(x \cdot f)}{\sum f}$$

# Mean, Median, Mode of a Freq. Dist. (Example)

**WEX 14-2-3:** Given the following frequency distribution:

DATA VALUE ( $x$ )	FREQUENCY ( $f$ )
<b>6</b>	<b>23</b>
<b>9</b>	<b>12</b>
<b>20</b>	<b>31</b>

- (a) Compute the mean of the frequency distribution.
- (b) Compute the median of the frequency distribution.
- (c) Compute the mode of the frequency distribution.

$$(a) \bar{x} = \frac{\sum(x \cdot f)}{\sum f} = \frac{(6)(23) + (9)(12) + (20)(31)}{23 + 12 + 31}$$

# Mean, Median, Mode of a Freq. Dist. (Example)

**WEX 14-2-3:** Given the following frequency distribution:

DATA VALUE ( $x$ )	FREQUENCY ( $f$ )
6	23
9	12
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- (a) Compute the mean of the frequency distribution.
- (b) Compute the median of the frequency distribution.
- (c) Compute the mode of the frequency distribution.

$$(a) \bar{x} = \frac{\sum(x \cdot f)}{\sum f} = \frac{(6)(23) + (9)(12) + (20)(31)}{23 + 12 + 31} = \frac{866}{66} \approx \boxed{13.1212}$$

# Mean, Median, Mode of a Freq. Dist. (Example)

**WEX 14-2-3:** Given the following frequency distribution:

DATA VALUE ( $x$ )	FREQUENCY ( $f$ )
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$$(b) \frac{\sum f}{2} = \frac{66}{2} = 33$$

# Mean, Median, Mode of a Freq. Dist. (Example)

**WEX 14-2-3:** Given the following frequency distribution:

DATA VALUE ( $x$ )	FREQUENCY ( $f$ )	LAST POSITION
6	23	23 <sup>rd</sup> position
<b>9</b>	12	23 + 12 = 35 <sup>th</sup> position
20	31	23 + 12 + 31 = 66 <sup>th</sup> position

- Compute the mean of the frequency distribution.
- Compute the median of the frequency distribution.
- Compute the mode of the frequency distribution.

$$(a) \bar{x} = \frac{\sum(x \cdot f)}{\sum f} = \frac{(6)(23) + (9)(12) + (20)(31)}{23 + 12 + 31} = \frac{866}{66} \approx \boxed{13.1212}$$

$$(b) \frac{\sum f}{2} = \frac{66}{2} = 33$$

$$\implies \text{Median} = (\text{Avg of } 33^{\text{rd}} \text{ Value \& } 34^{\text{th}} \text{ Value}) = \frac{9 + 9}{2} = \boxed{9}$$

# Mean, Median, Mode of a Freq. Dist. (Example)

**WEX 14-2-3:** Given the following frequency distribution:

DATA VALUE ( $x$ )	FREQUENCY ( $f$ )
6	23
9	12
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$$(b) \frac{\sum f}{2} = \frac{66}{2} = 33$$

$$\Rightarrow \text{Median} = (\text{Avg of } 33^{\text{rd}} \text{ Value \& } 34^{\text{th}} \text{ Value}) = \frac{9 + 9}{2} = \boxed{9}$$

# Mean, Median, Mode of a Freq. Dist. (Example)

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- (a) Compute the mean of the frequency distribution.  
(b) Compute the median of the frequency distribution.  
(c) Compute the mode of the frequency distribution.

$$(a) \bar{x} = \frac{\sum(x \cdot f)}{\sum f} = \frac{(6)(23) + (9)(12) + (20)(31)}{23 + 12 + 31} = \frac{866}{66} \approx \boxed{13.1212}$$

$$(b) \frac{\sum f}{2} = \frac{66}{2} = 33$$

$$\implies \text{Median} = (\text{Avg of } 33^{\text{rd}} \text{ Value \& } 34^{\text{th}} \text{ Value}) = \frac{9 + 9}{2} = \boxed{9}$$

- (c) The value with the highest frequency = **20**.



# Mean, Median, Mode of a Freq. Dist. (Example)

**WEX 14-2-3:** Given the following frequency distribution:

DATA VALUE ( $x$ )	FREQUENCY ( $f$ )
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- (a) Compute the mean of the frequency distribution.
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$$(b) \frac{\sum f}{2} = \frac{66}{2} = 33$$

$$\Rightarrow \text{Median} = (\text{Avg of } 33^{\text{rd}} \text{ Value \& } 34^{\text{th}} \text{ Value}) = \frac{9 + 9}{2} = \boxed{9}$$

- (c) The value with the highest frequency = 20.

$$\Rightarrow \text{Mode} = \boxed{20}$$

Fin.