Statistics: Range, Standard Deviation

Contemporary Math

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TTU

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Summarizing Data by Measuring its Dispersion

One way to summarize data is to **visualize** the data using:

- Bar Graphs
- Histrograms
- Stem-and-Leaf Displays

One way to numerically summarize data is to measure its **central tendency**:

- Mean
- Median
- Mode

A 2^{nd} way to numerically summarize data is to measure its **dispersion**:

- Range
- Standard Deviation

Range & Standard Deviation of a Data Set (Definition)

Definition

(Range of a Data Set)

Given a data set with n data values.
Then the **range** is the difference between the largest and smallest values.

Definition

(Standard Deviation of a Data Set)

Given a data set with n data values.
Then the **standard deviation** measures ”how spread out” the data is:

$$s = \sqrt{\frac{\sum (x - \overline{x})^2}{n - 1}}$$

Range & Standard Deviation of a Data Set (Example)

WEX 14-3-1: Given the following data set:

3, 15, 8, 11, 15

(a) Compute the range of the data set.
(b) Compute the mean of the data set.
(c) Compute the standard deviation of the data set.

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(b) $\bar{x} = \dfrac{\sum x}{n}$

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(a) Range = (Largest Data Value) $-$ (Smallest Data Value) $= 15 - 3 = \boxed{12}$

(b) $\bar{x} = \dfrac{\sum x}{n} = \dfrac{3 + 15 + 8 + 11 + 15}{5} = \dfrac{52}{5} = \boxed{10.4}$

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(c) $s = \sqrt{\dfrac{\sum (x - \bar{x})^2}{n - 1}}$

$$= \sqrt{\frac{(\mathbf{3} - \mathbf{10.4})^2 + (\mathbf{15} - \mathbf{10.4})^2 + (\mathbf{8} - \mathbf{10.4})^2 + (\mathbf{11} - \mathbf{10.4})^2 + (\mathbf{15} - \mathbf{10.4})^2}{5 - 1}}$$

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(c) $s = \sqrt{\dfrac{\sum (x - \bar{x})^2}{n - 1}}$

$$= \sqrt{\frac{(3 - 10.4)^2 + (15 - 10.4)^2 + (8 - 10.4)^2 + (11 - 10.4)^2 + (15 - 10.4)^2}{5 - 1}}$$

$$= \sqrt{\frac{103.2}{4}}$$

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(c) $s = \sqrt{\dfrac{\sum (x-\bar{x})^2}{n-1}}$

$= \sqrt{\dfrac{(3-10.4)^2 + (15-10.4)^2 + (8-10.4)^2 + (11-10.4)^2 + (15-10.4)^2}{5-1}}$

$= \sqrt{\dfrac{103.2}{4}} \approx \boxed{5.0794}$

Range & Standard Deviation of a Freq. Dist. (Def'n)

Definition

(Range of a Frequency Distribution)

The **range** is the difference between the largest and smallest values.

Definition

(Standard Deviation of a Frequency Distribution)

The **standard deviation** measures "how spread out" the data is:

$$s = \sqrt{\frac{\sum \left[(x - \bar{x})^2 \cdot f \right]}{\left(\sum f\right) - 1}}$$

WEX 14-3-2: Given the following frequency distribution:

DATA VALUE (x)	**FREQUENCY** (f)
6	23
9	12
20	31

(a) Compute the range of the frequency distribution.
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Range & Standard Dev. of a Freq. Dist. (Example)

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(b) $\bar{x} = \dfrac{\sum(x \cdot f)}{\sum f}$

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(b) $\bar{x} = \dfrac{\sum(x \cdot f)}{\sum f} = \dfrac{(\mathbf{6})(\mathbf{23}) + (\mathbf{9})(\mathbf{12}) + (\mathbf{20})(\mathbf{31})}{\mathbf{23} + \mathbf{12} + \mathbf{31}}$

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(b) $\bar{x} = \dfrac{\sum(x \cdot f)}{\sum f} = \dfrac{(6)(23) + (9)(12) + (20)(31)}{23 + 12 + 31} = \dfrac{866}{66} \approx \boxed{13.1212}$

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(c) $s = \sqrt{\dfrac{\sum\left[(x - \bar{x})^2 \cdot f\right]}{(\sum f) - 1}}$

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(c) $s = \sqrt{\dfrac{\sum\left[(x - \bar{x})^2 \cdot f\right]}{(\sum f) - 1}}$

$$= \sqrt{\frac{\mathbf{23}(\mathbf{6} - 13.1212)^2 + \mathbf{12}(\mathbf{9} - 13.1212)^2 + \mathbf{31}(\mathbf{20} - 13.1212)^2}{66 - 1}}$$

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(c) $s = \sqrt{\dfrac{\sum\left[(x - \bar{x})^2 \cdot f\right]}{(\sum f) - 1}}$

$= \sqrt{\dfrac{23(6 - 13.1212)^2 + 12(9 - 13.1212)^2 + 31(20 - 13.1212)^2}{66 - 1}}$

$= \sqrt{\dfrac{2837.03}{65}} = \boxed{6.607}$

Fin.