Logic: Statements, Connectives, Quantifiers

Contemporary Math

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Logic Statements (Definition)

Definition

(Logic Statements)

A **(logic) statement** is a **declarative sentence** that is **either true or false**.

Examples of Statements:

- "Janice works the morning shift."
- "I prefer cats to gerbils."
- "You expect the next iPad to be released within three years."
- Mathematical Statements: $1 + 1 = 2, \quad 4 - 5 = 7, \quad 4 \neq 5, \quad 9 \neq 9, \quad \ldots$

The following are **not** statements:

- Questions: "How much profit was made last quarter?"
- Commands: "Mow the front lawn."
- Exclamations: "Alright!", "Hey!", ...
- Onomatopoeia: "Tic, Toc", "Wham!", "Chirp!", "Woof!", ...
- Paradoxes: "This sentence is false."

Compound Statement & Connectives

Definition

(Simple Statement)

A **simple statement** contains a **single idea**.

Definition

(Compound Statement)

A **compound statement** contains **several simple statements (ideas)**. The ideas in a compounded statement are ”connected” by **connectives**. Moreover, the ideas can be represented by **variables**: $P, Q, R, \ldots$

CONNECTIVE NAME:	**NOTATION:**	**MEANING:**
Conjunction	$P \wedge Q$	P and Q
Disjunction	$P \vee Q$	P or Q
Negation	$\sim P$	not P
Conditional	$P \longrightarrow Q$	if P then Q
Biconditional	$P \longleftrightarrow Q$	P if and only if Q

<u>REMARK:</u> Then symbol $\equiv$ means ”represents”

Compound Statement & Connectives

WEX 3-1-1: Let $P \equiv$ "Roses are red", $Q \equiv$ "Violets are blue".
Express each symbolic statement in English:

(a) $P \wedge Q$

(b) $\sim P \vee \sim Q$

(c) $\sim P \longrightarrow Q$

(d) $P \longleftrightarrow \sim Q$

WEX 3-1-1: Let $P \equiv$ "Roses are red", $Q \equiv$ "Violets are blue".
Express each symbolic statement in English:

(a) $P \wedge Q$ $\boxed{\text{"Roses are red \textbf{and} violets are blue"}}$

(b) $\sim P \vee \sim Q$

(c) $\sim P \longrightarrow Q$

(d) $P \longleftrightarrow \sim Q$

WEX 3-1-1: Let $P \equiv$ "Roses are red", $Q \equiv$ "Violets are blue".
Express each symbolic statement in English:

(a) $P \wedge Q$ "Roses are red **and** violets are blue"

(b) $\sim P \vee \sim Q$ "Roses are **not** red **or** violets are **not** blue"

(c) $\sim P \longrightarrow Q$

(d) $P \longleftrightarrow \sim Q$

Compound Statement & Connectives

WEX 3-1-1: Let $P \equiv$ "Roses are red", $Q \equiv$ "Violets are blue".
Express each symbolic statement in English:

(a) $P \wedge Q$ $\boxed{\text{"Roses are red **and** violets are blue"}}$

(b) $\sim P \vee \sim Q$ $\boxed{\text{"Roses are **not** red **or** violets are **not** blue"}}$

(c) $\sim P \longrightarrow Q$ $\boxed{\text{"**If** roses are **not** red, **then** violets are blue"}}$

(d) $P \longleftrightarrow \sim Q$

Compound Statement & Connectives

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(b) $\sim P \vee \sim Q$ $\boxed{\text{"Roses are not red or violets are not blue"}}$

(c) $\sim P \longrightarrow Q$ $\boxed{\text{"If roses are not red, then violets are blue"}}$

(d) $P \longleftrightarrow \sim Q$ $\boxed{\text{"Roses are red if and only if violets are not blue"}}$

Quantifiers & Quantified Statements (Definition)

Definition

(Quantifiers)

Quantifiers express **how many "objects"** satisfy a given property or idea.
A **quantified statement** is a statement with **at least one quantifier**.

Universal Quantifiers: "All", "Every", "Each"
Existential Quantifiers: "Some", "At least one", "There exists", "There is/are"

WARNING: Quantifier "Any" can be either universal or existential!

Examples of **quantified statements**:

- "**All** roses are red", "**Every** rose is red", "**Each** rose is red"
- "**Some** violets are blue", "**At least one** violet is blue", "**There exists** a blue violet", "**There is** a blue violet", "**There are** blue violets"

Negation of Quantified Statements

Sometimes, the **negation** of a **quantified statement** must be considered:

QUANTIFIED STATEMENT	NEGATION
”Some....are....”	”No....are....”
”All....are....”	”Some....are not....”
”No....are....”	”Some....are....”
”Some....are not....”	”All....are....”

Negation of Quantified Statements (Example)

WEX 3-1-2: Negate the quantified statements:

(a) ”Some roses are red”

(b) ”All violets are blue”

(c) ”No violets are blue”

(d) ”Some roses are not red”

Negation of Quantified Statements (Example)

WEX 3-1-2: Negate the quantified statements:

(a) ”Some roses are red” $\boxed{\text{”No roses are red”}}$

(b) ”All violets are blue”

(c) ”No violets are blue”

(d) ”Some roses are not red”

Negation of Quantified Statements (Example)

WEX 3-1-2: Negate the quantified statements:

(a) "Some roses are red" $\boxed{\text{"No roses are red"}}$

(b) "All violets are blue" $\boxed{\text{"Some violets are not blue"}}$

(c) "No violets are blue"

(d) "Some roses are not red"

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(a) ”Some roses are red” $\boxed{\text{”No roses are red”}}$

(b) ”All violets are blue” $\boxed{\text{”Some violets are not blue”}}$

(c) ”No violets are blue” $\boxed{\text{”Some violets are blue”}}$

(d) ”Some roses are not red” $\boxed{\text{”All roses are red”}}$

Fin.