Logic: Truth Tables, DeMorgan's Laws

Contemporary Math

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TTU

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Truth Tables for the AND, OR, NOT Connectives

Truth Table for Conjunction (AND):

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Truth Table for Disjunction (OR):

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Truth Table for Negation (NOT):

P	$\sim P$
T	F
F	T

(Truth tables for the Conditional & Biconditional are seen in next section.)

Inclusive OR ($\vee$) versus Exclusive OR (**XOR**)

Disjunction is implicitly **inclusive OR**.

The truth table for **inclusive OR**:

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

The truth table for **exclusive OR**, is:

P	Q	P **XOR** Q
T	T	F
T	F	T
F	T	T
F	F	F

The difference in their truth tables is in blue.

Examples in English:

- Inclusive OR: "The car is compact or red" (or both compact and red)
- Exclusive OR: "I (either) drove to Austin or drove to Dallas" (but not both)

Logic Connectives (Order of Operations)

It's important to know the "order of operations" of logic connectives.
Otherwise, statements would require too many parentheses & brackets.

DOMINANCE:	CONNECTIVES:
MOST DOMINANT	Biconditional $\longleftrightarrow$
2^{nd} DOMINANT	Conditional $\longrightarrow$
3^{rd} DOMINANT	Conjunction $\wedge$ Disjunction $\vee$
LEAST DOMINANT	Negation $\sim$

REMARK: Since conjunction & disjunction has equal dominance, statements involving several of them require parentheses & square brackets!

For example:

- $P \wedge Q \vee R$ is ambiguous! It needs to change to one of the following:
  - ⋆ $(P \wedge Q) \vee R$
  - ⋆ $P \wedge (Q \vee R)$
  - ⋆ WARNING: The above two statements have **different truth tables**!
- $(\sim P \vee \sim Q) \wedge \sim R$ is equivalent to $[(\sim P) \vee (\sim Q)] \wedge (\sim R)$
- $(Q \wedge \sim P) \longrightarrow \sim R$ is equivalent to $[(Q \wedge (\sim P))] \longrightarrow (\sim R)$

Truth Tables (Example)

WEX 3-2-1: Construct a truth table for the logic statement: $\sim (P \wedge \sim Q)$

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Truth Tables (Example)

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P	Q	$\sim Q$	$P \wedge \sim Q$	$\sim(P \wedge \sim Q)$
T				
T				
F				
F				

WEX 3-2-1: Construct a truth table for the logic statement: $\sim (P \wedge \sim Q)$

P	Q	$\sim Q$	$P \wedge \sim Q$	$\sim (P \wedge \sim Q)$
T	T			
T	F			
F	T			
F	F			

Truth Tables (Example)

WEX 3-2-1: Construct a truth table for the logic statement: $\sim(P \wedge \sim Q)$

P	Q	$\sim Q$	$P \wedge \sim Q$	$\sim(P \wedge \sim Q)$
T	T	F		
T	F	T		
F	T	F		
F	F	T		

WEX 3-2-1: Construct a truth table for the logic statement: $\sim (P \wedge \sim Q)$

P	Q	$\sim Q$	$P \wedge \sim Q$	$\sim (P \wedge \sim Q)$
T	T	F	F	
T	F	T	T	
F	T	F	F	
F	F	T	F	

Truth Tables (Example)

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P	Q	$\sim Q$	$P \wedge \sim Q$	$\sim(P \wedge \sim Q)$
T	T	F	F	T
T	F	T	T	F
F	T	F	F	T
F	F	T	F	T

Truth Tables (Example)

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P	Q	$\sim Q$	$P \wedge \sim Q$	$\sim (P \wedge \sim Q)$
T	T	F	F	T
T	F	T	T	F
F	T	F	F	T
F	F	T	F	T

Truth Tables (Example)

WEX 3-2-1: Construct a truth table for the logic statement: $\sim(P \wedge \sim Q)$

P	Q	$\sim Q$	$P \wedge \sim Q$	$\sim(P \wedge \sim Q)$
T	**T**	F	F	**T**
T	F	T	T	F
F	T	F	F	T
F	F	T	F	T

While not required, let's do a quick a posteriori analysis of the truth table:

- If P is **true** and Q is **true**, then $\sim(P \wedge \sim Q)$ is **true**.

Truth Tables (Example)

WEX 3-2-1: Construct a truth table for the logic statement: $\sim (P \wedge \sim Q)$

P	Q	$\sim Q$	$P \wedge \sim Q$	$\sim (P \wedge \sim Q)$
T	T	F	F	T
T	**F**	T	T	**F**
F	T	F	F	T
F	F	T	F	T

While not required, let's do a quick a posteriori analysis of the truth table:

- If P is **true** and Q is **false**, then $\sim (P \wedge \sim Q)$ is **false**.

Truth Tables (Example)

WEX 3-2-1: Construct a truth table for the logic statement: $\sim(P \wedge \sim Q)$

P	Q	$\sim Q$	$P \wedge \sim Q$	$\sim(P \wedge \sim Q)$
T	T	F	F	T
T	F	T	T	F
F	**T**	F	F	**T**
F	F	T	F	T

While not required, let's do a quick a posteriori analysis of the truth table:

- If P is **false** and Q is **true**, then $\sim(P \wedge \sim Q)$ is **true**.

Truth Tables (Example)

WEX 3-2-1: Construct a truth table for the logic statement: $\sim (P \wedge \sim Q)$

P	Q	$\sim Q$	$P \wedge \sim Q$	$\sim (P \wedge \sim Q)$
T	T	F	F	T
T	F	T	T	F
F	T	F	F	T
F	**F**	T	F	**T**

While not required, let's do a quick a posteriori analysis of the truth table:

- If P is **false** and Q is **false**, then $\sim (P \wedge \sim Q)$ is **true**.

Truth Tables (Example)

WEX 3-2-2: Construct a truth table for the logic statement: $(P \wedge Q) \wedge \sim R$

Truth Tables (Example)

WEX 3-2-2: Construct a truth table for the logic statement: $(P \wedge Q) \wedge \sim R$

P	Q	R	$(P \wedge Q)$	$\sim R$	$(P \wedge Q) \wedge \sim R$

Truth Tables (Example)

WEX 3-2-2: Construct a truth table for the logic statement: $(P \wedge Q) \wedge \sim R$

P	Q	R	$(P \wedge Q)$	$\sim R$	$(P \wedge Q) \wedge \sim R$
T					
T					
T					
T					
F					
F					
F					
F					

Truth Tables (Example)

WEX 3-2-2: Construct a truth table for the logic statement: $(P \wedge Q) \wedge \sim R$

P	Q	R	$(P \wedge Q)$	$\sim R$	$(P \wedge Q) \wedge \sim R$
T	T				
T	T				
T	F				
T	F				
F	T				
F	T				
F	F				
F	F				

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T	T	T			
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
F	F	T			
F	F	F			

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P	Q	R	$(P \wedge Q)$	$\sim R$	$(P \wedge Q) \wedge \sim R$
T	T	T	T		
T	T	F	T		
T	F	T	F		
T	F	F	F		
F	T	T	F		
F	T	F	F		
F	F	T	F		
F	F	F	F		

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P	Q	R	$(P \wedge Q)$	$\sim R$	$(P \wedge Q) \wedge \sim R$
T	T	T	T	F	
T	T	F	T	T	
T	F	T	F	F	
T	F	F	F	T	
F	T	T	F	F	
F	T	F	F	T	
F	F	T	F	F	
F	F	F	F	T	

Truth Tables (Example)

WEX 3-2-2: Construct a truth table for the logic statement: $(P \wedge Q) \wedge \sim R$

P	Q	R	$(P \wedge Q)$	$\sim R$	$(P \wedge Q) \wedge \sim R$
T	T	T	T	F	F
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	F	T	F
F	T	T	F	F	F
F	T	F	F	T	F
F	F	T	F	F	F
F	F	F	F	T	F

Truth Tables (Example)

WEX 3-2-2: Construct a truth table for the logic statement: $(P \wedge Q) \wedge \sim R$

P	Q	R	$(P \wedge Q)$	$\sim R$	$(P \wedge Q) \wedge \sim R$
T	T	T	T	F	F
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	F	T	F
F	T	T	F	F	F
F	T	F	F	T	F
F	F	T	F	F	F
F	F	F	F	T	F

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WEX 3-2-2: Construct a truth table for the logic statement: $(P \wedge Q) \wedge \sim R$

P	Q	R	$(P \wedge Q)$	$\sim R$	$(P \wedge Q) \wedge \sim R$
T	**T**	**T**	T	F	**F**
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	F	T	F
F	T	T	F	F	F
F	T	F	F	T	F
F	F	T	F	F	F
F	F	F	F	T	F

While not required, let's do a quick a posteriori analysis of the truth table:

- If P is **true**, Q is **true**, and R is **true**, then $(P \wedge Q) \wedge \sim R$ is **false**.

Truth Tables (Example)

WEX 3-2-2: Construct a truth table for the logic statement: $(P \wedge Q) \wedge \sim R$

P	Q	R	$(P \wedge Q)$	$\sim R$	$(P \wedge Q) \wedge \sim R$
T	T	T	T	F	F
T	**T**	**F**	T	T	**T**
T	F	T	F	F	F
T	F	F	F	T	F
F	T	T	F	F	F
F	T	F	F	T	F
F	F	T	F	F	F
F	F	F	F	T	F

While not required, let's do a quick a posteriori analysis of the truth table:

- If P is **true**, Q is **true**, and R is **false**, then $(P \wedge Q) \wedge \sim R$ is **true**.

Truth Tables (Example)

WEX 3-2-2: Construct a truth table for the logic statement: $(P \wedge Q) \wedge \sim R$

P	Q	R	$(P \wedge Q)$	$\sim R$	$(P \wedge Q) \wedge \sim R$
T	T	T	T	F	F
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	F	T	F
F	T	T	F	F	F
F	**T**	**F**	F	T	**F**
F	F	T	F	F	F
F	F	F	F	T	F

While not required, let's do a quick a posteriori analysis of the truth table:

- If P is **false**, Q is **true**, and R is **false**, then $(P \wedge Q) \wedge \sim R$ is **false**.

Logical Equivalence (Definition)

Definition

(Logical Equivalence)

Two logic statements are **logically equivalent**, if they have the **same variables** (e.g. $P, Q, R, \ldots$) and the **final columns** in their **truth tables** are **identical**.

NOTATION: The symbol $\iff$ means "is logically equivalent to"

Logical Equivalence (Example)

WEX 3-2-3: Determine whether $P \wedge (Q \vee \sim R) \iff P \wedge \sim(\sim Q \wedge R)$ or not.

Logical Equivalence (Example)

<u>WEX 3-2-3:</u> Determine whether $P \wedge (Q \vee \sim R) \iff P \wedge \sim(\sim Q \wedge R)$ or not.

P	Q	R	$\sim R$	$Q \vee \sim R$	$P \wedge (Q \vee \sim R)$

Logical Equivalence (Example)

WEX 3-2-3: Determine whether $P \wedge (Q \vee \sim R) \iff P \wedge \sim(\sim Q \wedge R)$ or not.

P	Q	R	$\sim R$	$Q \vee \sim R$	$P \wedge (Q \vee \sim R)$
T					
T					
T					
T					
F					
F					
F					
F					

Logical Equivalence (Example)

WEX 3-2-3: Determine whether $P \wedge (Q \vee \sim R) \iff P \wedge \sim(\sim Q \wedge R)$ or not.

P	Q	R	$\sim R$	$Q \vee \sim R$	$P \wedge (Q \vee \sim R)$
T	T				
T	T				
T	F				
T	F				
F	T				
F	T				
F	F				
F	F				

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P	Q	R	$\sim R$	$Q \vee \sim R$	$P \wedge (Q \vee \sim R)$
T	T	T			
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
F	F	T			
F	F	F			

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P	Q	R	$\sim R$	$Q \vee \sim R$	$P \wedge (Q \vee \sim R)$
T	T	T	F		
T	T	F	T		
T	F	T	F		
T	F	F	T		
F	T	T	F		
F	T	F	T		
F	F	T	F		
F	F	F	T		

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P	Q	R	$\sim R$	$Q \vee \sim R$	$P \wedge (Q \vee \sim R)$
T	T	T	F	T	
T	T	F	T	T	
T	F	T	F	F	
T	F	F	T	T	
F	T	T	F	T	
F	T	F	T	T	
F	F	T	F	F	
F	F	F	T	T	

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P	Q	R	$\sim R$	$Q \vee \sim R$	$P \wedge (Q \vee \sim R)$
T	T	T	F	T	T
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	T	T	T
F	T	T	F	T	F
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F	F	F	T	T	F

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WEX 3-2-3: Determine whether $P \wedge (Q \vee \sim R) \iff P \wedge \sim (\sim Q \wedge R)$ or not.

P	Q	R	$\sim R$	$Q \vee \sim R$	$P \wedge (Q \vee \sim R)$
T	T	T	F	T	T
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	T	T	T
F	T	T	F	T	F
F	T	F	T	T	F
F	F	T	F	F	F
F	F	F	T	T	F

P	Q	R	$\sim Q$	$\sim Q \wedge R$	$\sim (\sim Q \wedge R)$	$P \wedge \sim (\sim Q \wedge R)$

Logical Equivalence (Example)

WEX 3-2-3: Determine whether $P \wedge (Q \vee \sim R) \iff P \wedge \sim(\sim Q \wedge R)$ or not.

P	Q	R	$\sim R$	$Q \vee \sim R$	$P \wedge (Q \vee \sim R)$
T	T	T	F	T	T
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	T	T	T
F	T	T	F	T	F
F	T	F	T	T	F
F	F	T	F	F	F
F	F	F	T	T	F

P	Q	R	$\sim Q$	$\sim Q \wedge R$	$\sim(\sim Q \wedge R)$	$P \wedge \sim(\sim Q \wedge R)$
T	T	T				
T	T	F				
T	F	T				
T	F	F				
F	T	T				
F	T	F				
F	F	T				
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P	Q	R	$\sim R$	$Q \vee \sim R$	$P \wedge (Q \vee \sim R)$
T	T	T	F	T	T
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	T	T	T
F	T	T	F	T	F
F	T	F	T	T	F
F	F	T	F	F	F
F	F	F	T	T	F

P	Q	R	$\sim Q$	$\sim Q \wedge R$	$\sim(\sim Q \wedge R)$	$P \wedge \sim(\sim Q \wedge R)$
T	T	T	F			
T	T	F	F			
T	F	T	T			
T	F	F	T			
F	T	T	F			
F	T	F	F			
F	F	T	T			
F	F	F	T			

Logical Equivalence (Example)

WEX 3-2-3: Determine whether $P \wedge (Q \vee \sim R) \iff P \wedge \sim(\sim Q \wedge R)$ or not.

P	Q	R	$\sim R$	$Q \vee \sim R$	$P \wedge (Q \vee \sim R)$
T	T	T	F	T	T
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	T	T	T
F	T	T	F	T	F
F	T	F	T	T	F
F	F	T	F	F	F
F	F	F	T	T	F

P	Q	R	$\sim Q$	$\sim Q \wedge R$	$\sim(\sim Q \wedge R)$	$P \wedge \sim(\sim Q \wedge R)$
T	T	T	F	F		
T	T	F	F	F		
T	F	T	T	T		
T	F	F	T	F		
F	T	T	F	F		
F	T	F	F	F		
F	F	T	T	T		
F	F	F	T	F		

Logical Equivalence (Example)

WEX 3-2-3: Determine whether $P \wedge (Q \vee \sim R) \iff P \wedge \sim (\sim Q \wedge R)$ or not.

P	Q	R	$\sim R$	$Q \vee \sim R$	$P \wedge (Q \vee \sim R)$
T	T	T	F	T	T
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	T	T	T
F	T	T	F	T	F
F	T	F	T	T	F
F	F	T	F	F	F
F	F	F	T	T	F

P	Q	R	$\sim Q$	$\sim Q \wedge R$	$\sim (\sim Q \wedge R)$	$P \wedge \sim (\sim Q \wedge R)$
T	T	T	F	F	T	
T	T	F	F	F	T	
T	F	T	T	T	F	
T	F	F	T	F	T	
F	T	T	F	F	T	
F	T	F	F	F	T	
F	F	T	T	T	F	
F	F	F	T	F	T	

Logical Equivalence (Example)

WEX 3-2-3: Determine whether $P \wedge (Q \vee \sim R) \iff P \wedge \sim(\sim Q \wedge R)$ or not.

P	Q	R	$\sim R$	$Q \vee \sim R$	$P \wedge (Q \vee \sim R)$
T	T	T	F	T	T
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	T	T	T
F	T	T	F	T	F
F	T	F	T	T	F
F	F	T	F	F	F
F	F	F	T	T	F

P	Q	R	$\sim Q$	$\sim Q \wedge R$	$\sim(\sim Q \wedge R)$	$P \wedge \sim(\sim Q \wedge R)$
T	T	T	F	F	T	T
T	T	F	F	F	T	T
T	F	T	T	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	F
F	T	F	F	F	T	F
F	F	T	T	T	F	F
F	F	F	T	F	T	F

Logical Equivalence (Example)

WEX 3-2-3: Determine whether $P \wedge (Q \vee \sim R) \iff P \wedge \sim(\sim Q \wedge R)$ or not.

P	Q	R	$\sim R$	$Q \vee \sim R$	$P \wedge (Q \vee \sim R)$
T	T	T	F	T	T
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	T	T	T
F	T	T	F	T	F
F	T	F	T	T	F
F	F	T	F	F	F
F	F	F	T	T	F

P	Q	R	$\sim Q$	$\sim Q \wedge R$	$\sim(\sim Q \wedge R)$	$P \wedge \sim(\sim Q \wedge R)$
T	T	T	F	F	T	T
T	T	F	F	F	T	T
T	F	T	T	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	F
F	T	F	F	F	T	F
F	F	T	T	T	F	F
F	F	F	T	F	T	F

Logical Equivalence (Example)

WEX 3-2-3: Determine whether $P \wedge (Q \vee \sim R) \iff P \wedge \sim (\sim Q \wedge R)$ or not.

P	Q	R	$\sim R$	$Q \vee \sim R$	$P \wedge (Q \vee \sim R)$	$P \wedge \sim (\sim Q \wedge R)$
T	T	T	F	T	T	T
T	T	F	T	T	T	T
T	F	T	F	F	F	F
T	F	F	T	T	T	T
F	T	T	F	T	F	F
F	T	F	T	T	F	F
F	F	T	F	F	F	F
F	F	F	T	T	F	F

Since the corresponding entries (in blue) of
$P \wedge (Q \vee \sim R)$ and $P \wedge \sim (\sim Q \wedge R)$ all match,

$P \wedge (Q \vee \sim R)$ and $P \wedge \sim (\sim Q \wedge R)$ are logically equivalent.

Simplifying Logic Statements (DeMorgan's Laws)

Theorem

(DeMorgan's Laws)

(a) $\sim (\sim P) \iff P$

(b) $\sim (P \wedge Q) \iff (\sim P) \vee (\sim Q)$

(c) $\sim (P \vee Q) \iff (\sim P) \wedge (\sim Q)$

Simplifying Logic Statements (DeMorgan's Laws)

Theorem

(DeMorgan's Laws)

(a) $\sim(\sim P) \iff P$

(b) $\sim(P \wedge Q) \iff (\sim P) \vee (\sim Q)$

(c) $\sim(P \vee Q) \iff (\sim P) \wedge (\sim Q)$

PROOF:

(a)

P	$\sim P$	$\sim(\sim P)$
T	F	T
F	T	F

Simplifying Logic Statements (DeMorgan's Laws)

Theorem

(DeMorgan's Laws)

(a) $\sim(\sim P) \iff P$

(b) $\sim(P \wedge Q) \iff (\sim P) \vee (\sim Q)$

(c) $\sim(P \vee Q) \iff (\sim P) \wedge (\sim Q)$

PROOF:

(b)

P	Q	$P \wedge Q$	$\sim(P \wedge Q)$	$\sim P$	$\sim Q$	$(\sim P) \vee (\sim Q)$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

Simplifying Logic Statements (DeMorgan's Laws)

Theorem

(DeMorgan's Laws)

(a) $\sim(\sim P) \iff P$

(b) $\sim(P \wedge Q) \iff (\sim P) \vee (\sim Q)$

(c) $\sim(P \vee Q) \iff (\sim P) \wedge (\sim Q)$

PROOF:

(c)

P	Q	$P \vee Q$	$\sim(P \vee Q)$	$\sim P$	$\sim Q$	$(\sim P) \wedge (\sim Q)$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

Simplifying Logic Statements (Example)

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&\iff (\sim P) \wedge \sim (\sim Q) \vee \sim (\sim R) && \text{[DeMorgan (b)]} \\
&\iff \boxed{(\sim P) \wedge Q \vee R} && \text{[DeMorgan (a)]}
\end{aligned}
$$

Negating English Statements (Example)

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Negation: $\sim (P \vee Q) \iff (\sim P) \wedge (\sim Q)$

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Let $P \equiv$ "Flowers are not red", $Q \equiv$ "John's truck has anti-lock brakes."

Then, "Flowers are not red or John's truck has anti-lock brakes." $\equiv P \lor Q$

Negation: $\sim (P \lor Q) \iff (\sim P) \land (\sim Q)$

$\iff$ "Flowers are red and John's truck does not have anti-lock brakes."

Fin.