# Logic: Conditional & Biconditional

**Contemporary Math** 

Josh Engwer

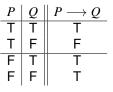
TTU

21 July 2015

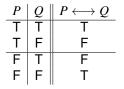
Josh Engwer (TTU)

#### Truth Tables for the Conditional & Biconditional

Truth Table for Conditional (IF...THEN):



Truth Table for Biconditional (IF AND ONLY IF):



## Logic Connectives (Order of Operations)

It's important to know the "order of operations" of logic connectives. Otherwise, statements would require too many parentheses & brackets.

DOMINANCE:	CONNECTIV	ES:
MOST DOMINANT	Biconditional	$\longleftrightarrow$
2 <sup>nd</sup> DOMINANT	Conditional	$\rightarrow$
3 <sup>rd</sup> DOMINANT	Conjunction	Λ
5 DOMINANT	Disjunction	V
LEAST DOMINANT	Negation	$\sim$

<u>REMARK:</u> Since conjunction & disjunction has equal dominance, statements involving several of them require parentheses & square brackets!

For example:

- *P* ∧ *Q* ∨ *R* is ambiguous! It needs to changed to one of the following:
   ★ (*P* ∧ *Q*) ∨ *R*
  - $\star P \wedge (Q \vee R)$
  - \* WARNING: The above two statements have different truth tables!
- (~ P ∨ ~ Q) ∧ ~ R is equivalent to
  (Q ∧ ~ P) → ~ R is equivalent to

t to 
$$[(\sim P) \lor (\sim Q)] \land (\sim R)$$
  
t to  $[(Q \land (\sim P))] \longrightarrow (\sim R)$ 

Р	Q	R	$\sim P$	$\sim P \longrightarrow Q$	$\sim R$	$(\sim P \longrightarrow Q) \longrightarrow \sim R$
						1

Р	Q	R	$\sim P$	$\sim P \longrightarrow Q$	$ \sim R $	$  (\sim P \longrightarrow Q) \longrightarrow \sim R$
Т						
Т						
Т						
Т						
F						
F						
F						
F						

**WEX 3-3-1:** Construct a truth table for the statement:  $(\sim P \longrightarrow Q) \longrightarrow \sim R$ 

Р	Q	R	$\sim P$	$\sim P \longrightarrow Q$	$ \sim R $	$  (\sim P \longrightarrow Q) \longrightarrow \sim R$
Т	Т					
Т	Т					
Т	F					
Т	F					
F	Т					
F	Т					
F	F					
F	F					

Р	Q	R	$\sim P$	$\sim P \longrightarrow Q$	$\sim R$	$  (\sim P \longrightarrow Q) \longrightarrow \sim R$
Т	Т	Т				
Т	Т	F				
Т	F	Т				
Т	F	F				
F	Т	Т				
F	Т	F				
F	F	Т				
F	F	F				

Р	Q	R	$\sim P$	$\sim P \longrightarrow Q$	$ \sim R $	$  (\sim P \longrightarrow Q) \longrightarrow \sim R$
Т	Т	Т	F			
Т	Т	F	F			
Т	F	Т	F			
Т	F	F	F			
F	Т	Т	Т			
F	Т	F	Т			
F	F	Т	Т			
F	F	F	Т			

Р	Q	R	$\sim P$	$\sim P \longrightarrow Q$	$\sim R$	$  (\sim P \longrightarrow Q) \longrightarrow \sim R$
Т	Т	Т	F	Т		
Т	Т	F	F	Т		
Т	F	Т	F	Т		
Т	F	F	F	Т		
F	Т	Т	Т	Т		
F	Т	F	Т	Т		
F	F	Т	Т	F		
F	F	F	Т	F		

Р	Q	R	$\sim P$	$\sim P \longrightarrow Q$	$\sim R$	$  (\sim P \longrightarrow Q) \longrightarrow \sim R$
Т	Т	Т	F	Т	F	
Т	Т	F	F	Т	Т	
Т	F	Т	F	Т	F	
Т	F	F	F	Т	Т	
F	Т	Т	Т	Т	F	
F	Т	F	Т	Т	Т	
F	F	Т	Т	F	F	
F	F	F	Т	F	T	

Р	Q	R	$\sim P$	$\sim P \longrightarrow Q$	$\sim R$	$  (\sim P \longrightarrow Q) \longrightarrow \sim R$
Т	Т	Т	F	Т	F	F
Т	Т	F	F	Т	Т	Т
Т	F	Т	F	Т	F	F
	•	F	F	т	Т	Т
F	Т	Т	Т	Т	F	F
F	Т	F	Т	Т	Т	Т
F	F	Т	Т	F	F	Т
F	F	F	Т	F	Т	Т

Р	Q	R	$\sim P$	$\sim P \longrightarrow Q$	$\sim R$	$(\sim P \longrightarrow Q) \longrightarrow \sim R$
Т	Т	Т	F	Т	F	F
Т	Т	F	F	Т	Т	Т
Т	F	T	F	Т	F	F
Т	F	F	F	Т	Т	Т
F	Т	Т	Т	Т	F	F
F	Т	F	Т	Т	Т	T
F	F	Т	Т	F	F	Т
F	F	F	Т	F	T	Т

#### **<u>WEX 3-3-2</u>**: Construct a truth table for: $\sim (P \land \sim Q) \longleftrightarrow P \lor \sim Q$

**<u>WEX 3-3-2</u>**: Construct a truth table for:  $\sim (P \land \sim Q) \longleftrightarrow P \lor \sim Q$ 

Р	Q	$\sim Q$	$P \wedge \sim Q$	$\sim (P \land \sim Q)$	$P \lor \sim Q$	$ \sim (P \land \sim Q) \longleftrightarrow P \lor \sim Q$

**WEX 3-3-2:** Construct a truth table for:  $\sim (P \land \sim Q) \longleftrightarrow P \lor \sim Q$ 

Р	Q	$\sim Q$	$P \wedge \sim Q$	$\sim (P \land \sim Q)$	$P \lor \sim Q$	$ \sim (P \land \sim Q) \longleftrightarrow P \lor \sim Q$
Т						
Т						
F						
F						

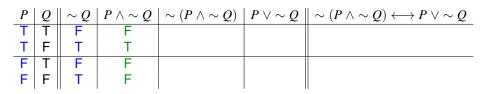
**<u>WEX 3-3-2</u>**: Construct a truth table for:  $\sim (P \land \sim Q) \longleftrightarrow P \lor \sim Q$ 

Р	Q	$\sim Q$	$P \wedge \sim Q$	$\sim (P \land \sim Q)$	$P \lor \sim Q$	$ \sim (P \wedge \sim Q) \longleftrightarrow P \lor \sim Q$
Т	Т					
Т	F					
F	Т					
F	F					

**WEX 3-3-2:** Construct a truth table for:  $\sim (P \land \sim Q) \longleftrightarrow P \lor \sim Q$ 

Р	Q	$\sim Q$	$P \wedge \sim Q$	$\sim (P \land \sim Q)$	$P \lor \sim Q$	$ \sim (P \land \sim Q) \longleftrightarrow P \lor \sim Q$
Т	Т	F				
Т	F	Т				
F		F				
F	F	Т				

**<u>WEX 3-3-2</u>**: Construct a truth table for:  $\sim (P \land \sim Q) \leftrightarrow P \lor \sim Q$ 



**<u>WEX 3-3-2</u>**: Construct a truth table for:  $\sim (P \land \sim Q) \longleftrightarrow P \lor \sim Q$ 

Р	Q	$\sim Q$	$P \wedge \sim Q$	$\sim (P \land \sim Q)$	$P \lor \sim Q$	$ \sim (P \land \sim Q) \longleftrightarrow P \lor \sim Q$
Т	Т	F	F	Т		
Т	F	Т	Т	F		
F	Т	F	F	Т		
F	F	Т	F	Т		

**<u>WEX 3-3-2</u>**: Construct a truth table for:  $\sim (P \land \sim Q) \longleftrightarrow P \lor \sim Q$ 

Р	Q	$\sim Q$	$P \wedge \sim Q$	$\sim (P \land \sim Q)$	$P \lor \sim Q$	$ \sim (P \land \sim Q) \longleftrightarrow P \lor \sim Q$
Т	Т	F	F	Т	Т	
Т	F	Т	Т	F	Т	
F	Т	F	F	Т	F	
F	F	Т	F	Т	Т	

**<u>WEX 3-3-2</u>**: Construct a truth table for:  $\sim (P \land \sim Q) \longleftrightarrow P \lor \sim Q$ 

Р	Q	$\sim Q$	$P \wedge \sim Q$	$\sim (P \land \sim Q)$	$P \lor \sim Q$	$ \sim (P \wedge \sim Q) \longleftrightarrow P \lor \sim Q$
Т	Т	F	F	Т	Т	Т
Т	F	Т	Т	F	Т	F
F	Т	F	F F	Т	F	F
F	F	Т	F	Т	Т	Т

**WEX 3-3-2:** Construct a truth table for:  $\sim (P \land \sim Q) \longleftrightarrow P \lor \sim Q$ 

Р	Q	$\sim Q$	$P \wedge \sim Q$	$\sim (P \land \sim Q)$	$P \lor \sim Q$	$ \sim (P \land \sim Q) \longleftrightarrow P \lor \sim Q_{-}$
Т	Т	F	F	Т	Т	Т
Т	F	Т	Т	F	Т	F
F	Т	F	F	Т	F	F
F	F	Т	F	Т	Т	Т

In English, conditionals can be worded various ways:

 $P \longrightarrow Q$ "If P, then Q" "Q if P" "P only if Q" "P is sufficient for Q" "Q is necessary for P"

#### Definition

(More about the Conditional)

Given the **conditional**  $P \longrightarrow Q$ ,

*P* is sometimes known as the **hypothesis** (or **antecedent**) *Q* is sometimes known as the **conclusion** (or **consequent**)

#### Definition

(Converses, Inverses, Contrapositives)

The	converse	of conditional $P \longrightarrow Q$	is	$Q \longrightarrow P$
The	inverse	of conditional $P \longrightarrow Q$	is	$\sim P \longrightarrow \sim Q$
The	contrapositive	of conditional $P \longrightarrow Q$	is	$\sim Q \longrightarrow \sim P$

#### Proposition

(Logical Equivalence w.r.t. Conditionals)

(a) 
$$\sim Q \longrightarrow \sim P \iff P \longrightarrow Q$$

$$(b) \quad \sim P \longrightarrow \sim Q \iff Q \longrightarrow P$$

# Converses, Inverses, Contrapositives of Conditionals

#### Definition

(Converses, Inverses, Contrapositives)

The	converse	of conditional $P \longrightarrow Q$	is	$Q \longrightarrow P$
The	inverse	of conditional $P \longrightarrow Q$	is	$\sim P \longrightarrow \sim Q$
The	contrapositive	of conditional $P \longrightarrow Q$	is	$\sim Q \longrightarrow \sim P$

#### Proposition

(Logical Equivalence w.r.t. Conditionals)

(a) 
$$\sim Q \longrightarrow \sim P \iff P \longrightarrow Q$$

$$(b) \quad \sim P \longrightarrow \sim Q \iff Q \longrightarrow P$$

# Converses, Inverses, Contrapositives of Conditionals

#### Definition

(Converses, Inverses, Contrapositives)

The	converse	of conditional $P \longrightarrow Q$	is	$Q \longrightarrow P$
The	inverse	of conditional $P \longrightarrow Q$	is	$\sim P \longrightarrow \sim Q$
The	contrapositive	of conditional $P \longrightarrow Q$	is	$\sim Q \longrightarrow \sim P$

#### Proposition

(Logical Equivalence w.r.t. Conditionals)

(a) 
$$\sim Q \longrightarrow \sim P \iff P \longrightarrow Q$$

$$(b) \quad \sim P \longrightarrow \sim Q \iff Q \longrightarrow P$$

- (a) Find the converse.
- (b) Find the inverse.
- (c) Find the contraposition.

Let  $P \equiv$  "Roses are red",  $Q \equiv$  "Violets are blue"

- (a) Find the converse.
- (b) Find the inverse.
- (c) Find the contraposition.

Let  $P \equiv$  "Roses are red",  $Q \equiv$  "Violets are blue" Then, "If roses are red, then violets are blue"  $\equiv P \longrightarrow Q$ 

- (a) Find the converse.
- (b) Find the inverse.
- (c) Find the contraposition.

Let  $P \equiv$  "Roses are red",  $Q \equiv$  "Violets are blue" Then, "If roses are red, then violets are blue"  $\equiv P \longrightarrow Q$ 

(a) Find the converse.

 $Q \longrightarrow P$ 

(b) Find the inverse.

 $\sim P \longrightarrow \sim Q$ 

(c) Find the contraposition.

 $\sim Q \longrightarrow \sim P$ 

Let  $P \equiv$  "Roses are red",  $Q \equiv$  "Violets are blue" Then, "If roses are red, then violets are blue"  $\equiv P \longrightarrow Q$ 

(a) Find the converse.

 $Q \longrightarrow P \iff$  "If violets are blue, then roses are red"

(b) Find the inverse.

 $\sim P \longrightarrow \sim Q \iff$  "If roses are not red, then violets are not blue"

(c) Find the contraposition.

 $\sim Q \longrightarrow \sim P \iff$  "If violets are not blue, then roses are not red"

# Fin.