Logic: Conditional & Biconditional

Contemporary Math

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TTU

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Truth Tables for the Conditional & Biconditional

Truth Table for Conditional (IF...THEN):

P	Q	$P \longrightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

Truth Table for Biconditional (IF AND ONLY IF):

P	Q	$P \longleftrightarrow Q$
T	T	T
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Logic Connectives (Order of Operations)

It's important to know the "order of operations" of logic connectives.
Otherwise, statements would require too many parentheses & brackets.

DOMINANCE:	CONNECTIVES:
MOST DOMINANT	Biconditional $\longleftrightarrow$
2^{nd} DOMINANT	Conditional $\longrightarrow$
3^{rd} DOMINANT	Conjunction $\wedge$ Disjunction $\vee$
LEAST DOMINANT	Negation $\sim$

REMARK: Since conjunction & disjunction has equal dominance, statements involving several of them require parentheses & square brackets!

For example:

- $P \wedge Q \vee R$ is ambiguous! It needs to changed to one of the following:
  - ⋆ $(P \wedge Q) \vee R$
  - ⋆ $P \wedge (Q \vee R)$
  - ⋆ WARNING: The above two statements have **different truth tables**!
- $(\sim P \vee \sim Q) \wedge \sim R$ is equivalent to $[(\sim P) \vee (\sim Q)] \wedge (\sim R)$
- $(Q \wedge \sim P) \longrightarrow \sim R$ is equivalent to $[(Q \wedge (\sim P))] \longrightarrow (\sim R)$

Truth Table involving the Conditional (Example)

WEX 3-3-1: Construct a truth table for the statement: $(\sim P \longrightarrow Q) \longrightarrow \sim R$

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More about Conditionals....

In English, **conditionals** can be worded various ways:

$P \longrightarrow Q$
"If P, then Q"
"Q if P"
"P only if Q"
"P is sufficient for Q"
"Q is necessary for P"

Definition

(More about the Conditional)

Given the **conditional** $P \longrightarrow Q$,

P is sometimes known as the **hypothesis** (or **antecedent**)
Q is sometimes known as the **conclusion** (or **consequent**)

Converses, Inverses, Contrapositives of Conditionals

Definition

(Converses, Inverses, Contrapositives)

The	**converse**	of conditional $P \longrightarrow Q$	is	$Q \longrightarrow P$
The	**inverse**	of conditional $P \longrightarrow Q$	is	$\sim P \longrightarrow \sim Q$
The	**contrapositive**	of conditional $P \longrightarrow Q$	is	$\sim Q \longrightarrow \sim P$

Proposition

(Logical Equivalence w.r.t. Conditionals)

(a) $\sim Q \longrightarrow \sim P \iff P \longrightarrow Q$

(b) $\sim P \longrightarrow \sim Q \iff Q \longrightarrow P$

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PROOF: (a)

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(Logical Equivalence w.r.t. Conditionals)

(a) $\sim Q \longrightarrow \sim P \iff P \longrightarrow Q$

(b) $\sim P \longrightarrow \sim Q \iff Q \longrightarrow P$

PROOF: (b)

P	Q	$\sim P$	$\sim Q$	$\sim P \longrightarrow \sim Q$	$Q \longrightarrow P$
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Converses, Inverses, Contrapositives of Conditionals

<u>**WEX 3-3-3:**</u> Given the statement ”If roses are red, then violets are blue”:

(a) Find the converse.

(b) Find the inverse.

(c) Find the contraposition.

Converses, Inverses, Contrapositives of Conditionals

WEX 3-3-3: Given the statement ”If roses are red, then violets are blue”:

Let $P \equiv$ ”Roses are red”, $Q \equiv$ ”Violets are blue”

(a) Find the converse.

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WEX 3-3-3: Given the statement "If roses are red, then violets are blue":

Let $P \equiv$ "Roses are red", $Q \equiv$ "Violets are blue"
Then, "If roses are red, then violets are blue" $\equiv P \longrightarrow Q$

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(a) Find the converse.

$Q \longrightarrow P$

(b) Find the inverse.

$\sim P \longrightarrow \sim Q$

(c) Find the contraposition.

$\sim Q \longrightarrow \sim P$

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Then, "If roses are red, then violets are blue" $\equiv P \longrightarrow Q$

(a) Find the converse.

$Q \longrightarrow P \iff$ "If violets are blue, then roses are red"

(b) Find the inverse.

$\sim P \longrightarrow \sim Q \iff$ "If roses are not red, then violets are not blue"

(c) Find the contraposition.

$\sim Q \longrightarrow \sim P \iff$ "If violets are not blue, then roses are not red"

Fin

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