Logic: Verifying Arguments

Contemporary Math

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Valid Arguments (Definition)

Arguments are prevalent in everyday life, law, math, science, etc...

Definition

(Argument)

An **argument** is a series of statements called **premises** followed by a single statement called the **conclusion**.

Example argument:

$$\begin{array}{ll} \text{If I think, then I am} & (1^{st} Premise) \\ \text{I think} & (2^{nd} Premise) \\ \hline \therefore \quad \text{I am} & (Conclusion) \end{array} \iff \begin{array}{ll} P \longrightarrow Q & (1^{st} Premise) \\ P & (2^{nd} Premise) \\ \hline \therefore \quad Q & (Conclusion) \end{array}$$

Definition

(Validity of an Argument)

An argument is **valid** if whenever all the premises are true, then the conclusion must also be true.

REMARK: The **form** of an argument is paramount here, not the content!

Common Valid Arguments

Certain fundamental valid arguments occur over and over again:

- Law of Detachment:
$$\begin{array}{l} P \longrightarrow Q \\ P \\ \hline \therefore \quad Q \end{array}$$

- Law of Contraposition:
$$\begin{array}{l} P \longrightarrow Q \\ \sim Q \\ \hline \therefore \quad \sim P \end{array}$$

- Law of Syllogism:
$$\begin{array}{l} P \longrightarrow Q \\ Q \longrightarrow R \\ \hline \therefore \quad P \longrightarrow R \end{array}$$

- Disjunctive Syllogism:
$$\begin{array}{l} P \vee Q \\ \sim P \\ \hline \therefore \quad Q \end{array} \qquad \begin{array}{l} P \vee Q \\ \sim Q \\ \hline \therefore \quad P \end{array}$$

Common Fallacies

Definition

(Fallacy)

A **fallacy** is a fundamental invalid argument.

- Fallacy of the Converse:
$$\begin{array}{l} P \longrightarrow Q \\ Q \\ \hline \therefore \ P \end{array}$$

- Fallacy of the Inverse:
$$\begin{array}{l} P \longrightarrow Q \\ \sim P \\ \hline \therefore \ Q \end{array}$$

- Affirming a Disjunction:
$$\begin{array}{l} P \vee Q \\ P \\ \hline \therefore \ \sim Q \end{array} \qquad \begin{array}{l} P \vee Q \\ Q \\ \hline \therefore \ \sim P \end{array}$$

Validity of an Argument (Example)

WEX 3-4-1: (a) What is the form of the following argument?
(b) Is the argument valid?

If gasoline is expensive, then there is less traffic.
Gasoline is expensive.

$\therefore$ There is less traffic.

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(a) The form of the argument is the **Law of Detachment**

(b) Since the Law of Detachment is a valid argument, the argument is **valid**

Validity of an Argument (Example)

WEX 3-4-2: (a) What is the form of the following argument?
(b) Is the argument valid?

If gasoline is expensive, then there is less traffic.
There is less traffic.

$\therefore$ Gasoline is expensive.

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There is less traffic.
$\therefore$ Gasoline is expensive.

$\Longleftrightarrow$

$P \longrightarrow Q$
Q
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(a) The form of the argument is the **Fallacy of the Converse**

(b) Since fallacies are invalid arguments, the argument is **invalid**

Verifying an Argument (Example)

WEX 3-4-3: Using a truth table, is the following argument valid or invalid?

$$\begin{array}{l} \sim P \longrightarrow \sim Q \\ Q \\ \hline \therefore \quad P \end{array}$$

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				PREMISES		CONCLUSION
P	Q	$\sim P$	$\sim Q$	Q	$\sim P \longrightarrow \sim Q$	P

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P	Q	$\sim P$	$\sim Q$	Q	$\sim P \longrightarrow \sim Q$	P
T	T					
T	F					
F	T					
F	F					

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T	T	F	F			
T	F	F	T			
F	T	T	F			
F	F	T	T			

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T	T	F	F	T	T	T
T	F	F	T	F	T	T
F	T	T	F	T	F	F
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T	T	F	F	**T**	**T**	**T**
T	F	F	T	F	T	T
F	T	T	F	T	F	F
F	F	T	T	F	T	F

Since every row where the premises are all true (in blue) also has the conclusion true (in green), The argument is **valid**

Verifying an Argument (Example)

WEX 3-4-4: Using a truth table, is the following argument valid or invalid?

$$\begin{array}{l} P \longrightarrow Q \\ \sim R \longrightarrow Q \\ \hline \therefore \quad P \wedge R \longrightarrow \sim Q \end{array}$$

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						PREMISES		**CONCLUSION**
P	Q	R	$\sim Q$	$\sim R$	$P \wedge R$	$P \longrightarrow Q$	$\sim R \longrightarrow Q$	$P \wedge R \longrightarrow \sim Q$

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T	T	T						
T	T	F						
T	F	T						
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P	Q	R	$\sim Q$	$\sim R$	$P \wedge R$	$P \longrightarrow Q$	$\sim R \longrightarrow Q$	$P \wedge R \longrightarrow \sim Q$
T	T	T	F	F	T			
T	T	F	F	T	F			
T	F	T	T	F	T			
T	F	F	T	T	F			
F	T	T	F	F	F			
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F	F	T	T	F	F			
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P	Q	R	$\sim Q$	$\sim R$	$P \wedge R$	$P \longrightarrow Q$	$\sim R \longrightarrow Q$	$P \wedge R \longrightarrow \sim Q$
T	T	T	F	F	T	T	T	F
T	T	F	F	T	F	T	T	T
T	F	T	T	F	T	F	T	T
T	F	F	T	T	F	F	F	T
F	T	T	F	F	F	T	T	T
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P	Q	R	$\sim Q$	$\sim R$	$P \wedge R$	$P \longrightarrow Q$	$\sim R \longrightarrow Q$	$P \wedge R \longrightarrow \sim Q$
T	T	T	F	F	T	**T**	**T**	**F**
T	T	F	F	T	F	**T**	**T**	**T**
T	F	T	T	F	T	F	T	T
T	F	F	T	T	F	F	F	T
F	T	T	F	F	F	**T**	**T**	**T**
F	T	F	F	T	F	**T**	**T**	**T**
F	F	T	T	F	F	**T**	**T**	**T**
F	F	F	T	T	F	T	F	T

Since there's at least one row where the premises are all true (in blue) but the conclusion is false (in red), The argument is **invalid**

Fin

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