Logic: Verifying Quantified Arguments

Contemporary Math

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TTU

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Quantified Arguments (Definition)

Last section involved verifying **arguments**.
Now, let's consider **arguments with quantifiers**.

Definition

(Quantified Argument)

A **quantified argument** is an argument with **at least one quantifier**.

Another name for quantified argument is **syllogism**.

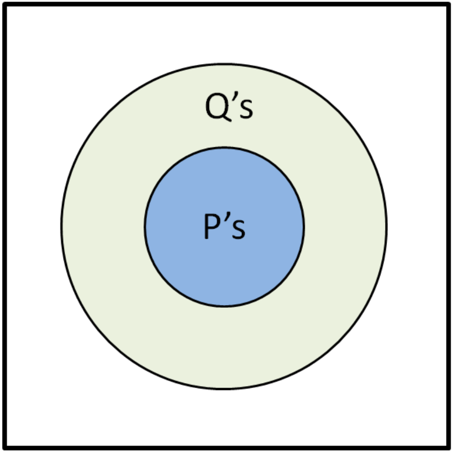
Example quantified argument:

All people have a phone.
Phil is a person.

$\therefore$ Phil has a phone.

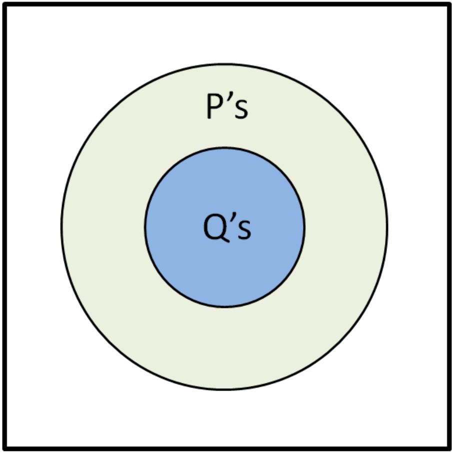
Verifying a **quantified** argument involves drawing an **Euler diagram**.

Quantifiers via Euler Diagrams (with 2 sets)



All P's are Q's. $\iff$ **Every** P is a Q. $\iff$ **Each** P is a Q.

Quantifiers via Euler Diagrams (with 2 sets)



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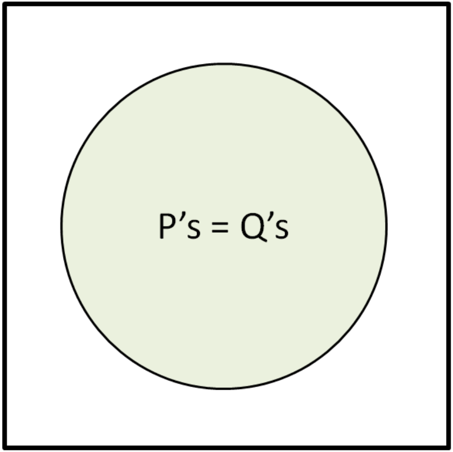
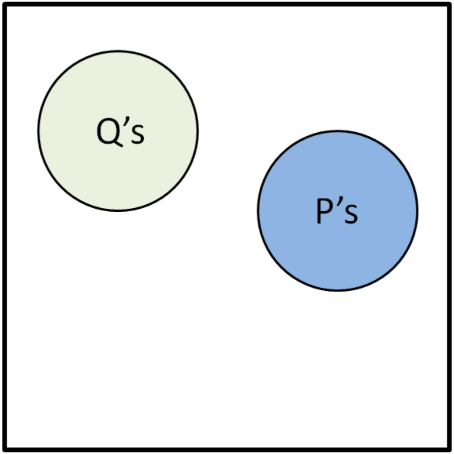


All P's are Q's.
All Q's are P's.

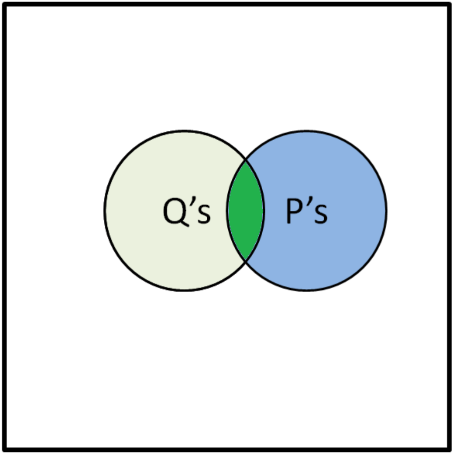
In this case, sets P & Q are **coincident** (**equal**)

Quantifiers via Euler Diagrams (with 2 sets)



No P's are Q's.
No Q's are P's.

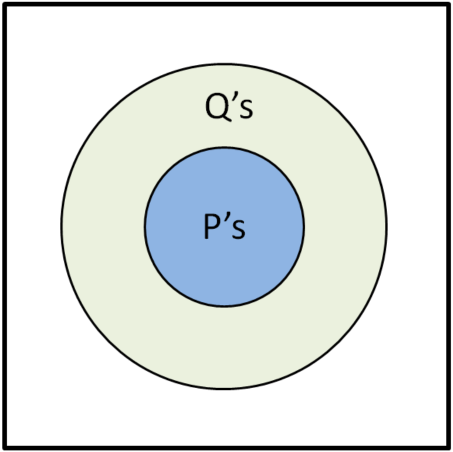
Quantifiers via Euler Diagrams (with 2 sets)



Some P's are Q's. (in green)
Some Q's are P's. (in green)

Some P's are **not** Q's. (in blue)
Some Q's are **not** P's. (in beige)

Quantifiers via Euler Diagrams (with 2 sets)



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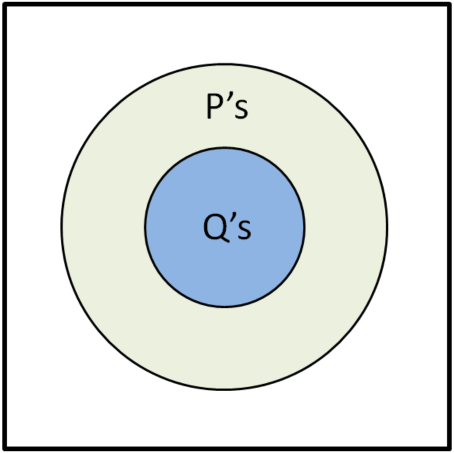

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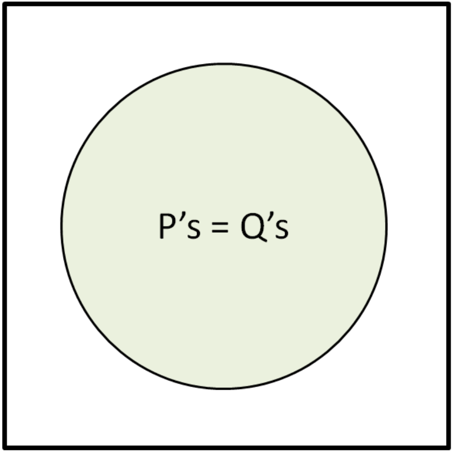

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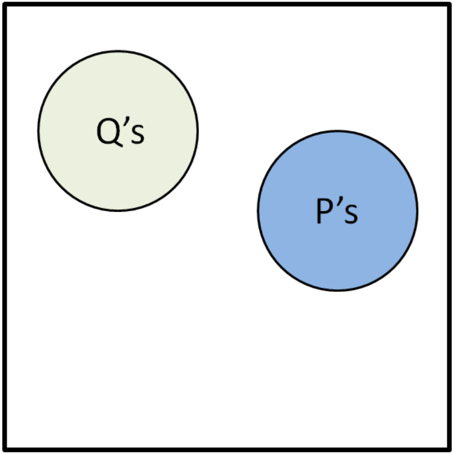

Some P's are **not** Q's.
Some Q's are **not** P's.

Euler Diagrams (Example)

WEX 3-5-1:

Using Euler Diagram(s), determine whether this argument is valid or not:

All people have a phone.
Phil is a person.

$\therefore$ Phil has a phone.

Euler Diagrams (Example)

<u>WEX 3-5-1:</u>

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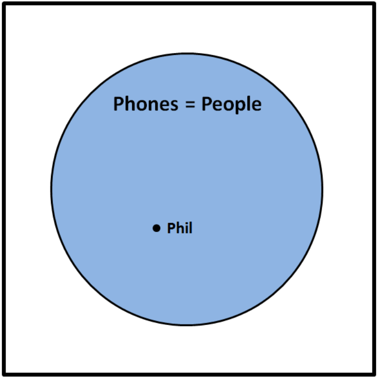


All people have a phone.
Phil is a person.

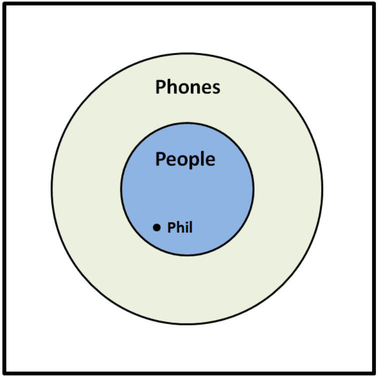
$\therefore$ Phil has a phone.

CASE I: The set of all phones is exactly equal to the set of all people

Euler Diagrams (Example)

<u>WEX 3-5-1:</u>

Using Euler Diagram(s), determine whether this argument is valid or not:



All people have a phone.
Phil is a person.

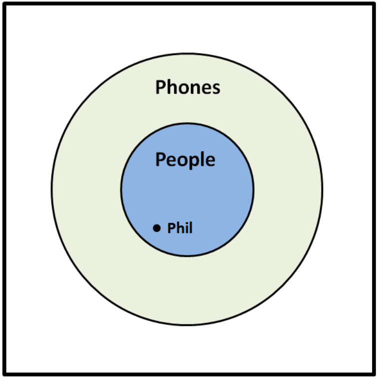
$\therefore$ Phil has a phone.

CASE II: The set of all phones contains the set of all people

Euler Diagrams (Example)

WEX 3-5-1:

Using Euler Diagram(s), determine whether this argument is valid or not:



All people have a phone.
Phil is a person.

$\therefore$ Phil has a phone.

In all possible cases, Phil is **always** inside the set of all phones.

Hence, the argument is **valid**

With quantified arguments with **3 sets**,
there are far too many possibilities to show here!!

Hence, what follows are two cases to illustrate some of these possibilities.

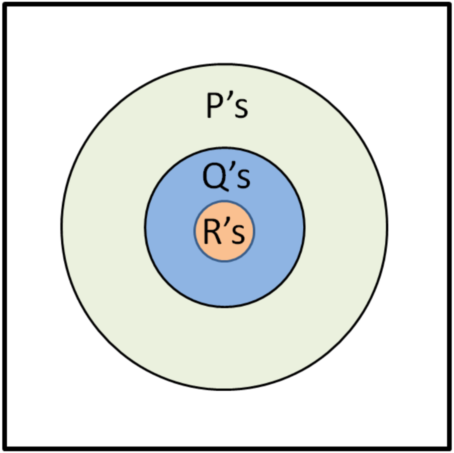
Quantifiers via Euler Diagrams (with 3 sets)

PART I:

All Q's are P's.
All R's are Q's.

$\therefore$????

Quantifiers via Euler Diagrams (with 3 sets)



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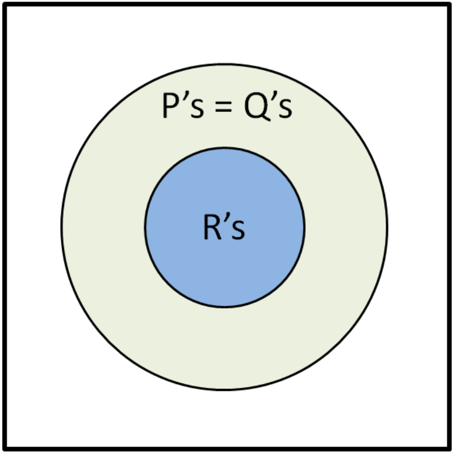

All Q's are P's.
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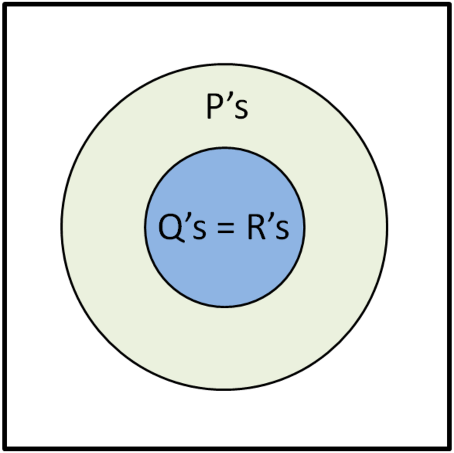
$\therefore$????

In this case, sets P & Q are **coincident** (**equal**)

Quantifiers via Euler Diagrams (with 3 sets)

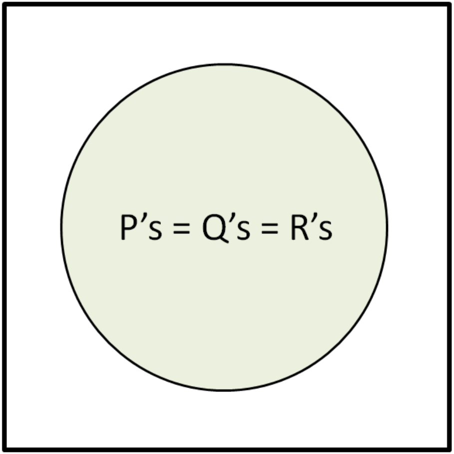


All Q's are P's.
All R's are Q's.

$\therefore$????

In this case, sets Q & R are **coincident** (**equal**)

Quantifiers via Euler Diagrams (with 3 sets)



All Q’s are P’s.
All R’s are Q’s.

$\therefore$????

In this case, all three sets P, Q, R are **coincident** (**equal**)

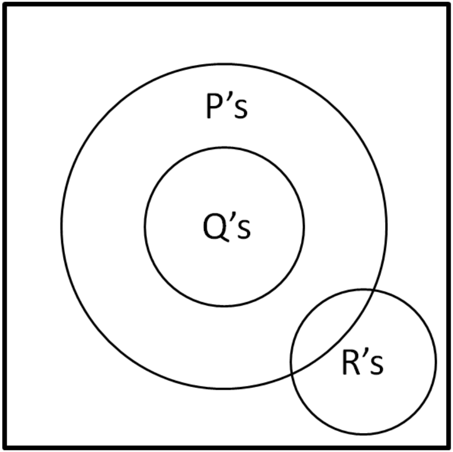
Quantifiers via Euler Diagrams (with 3 sets)

PART II:

All Q's are P's.
Some P's are R's.

$\therefore$????

Quantifiers via Euler Diagrams (with 3 sets)



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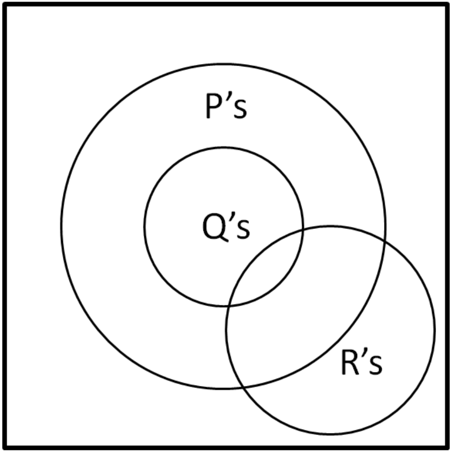

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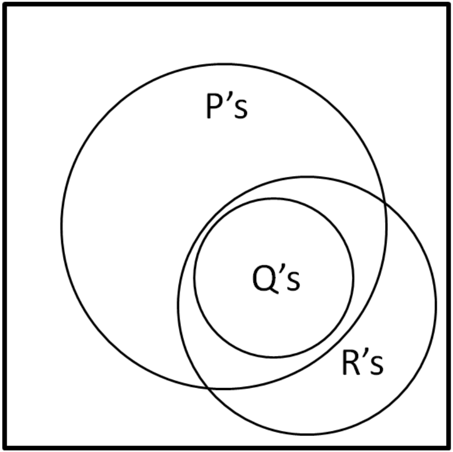

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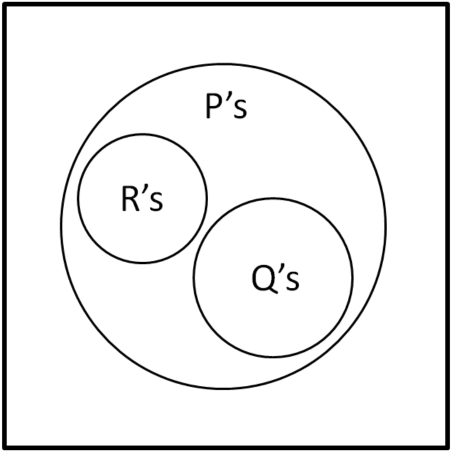

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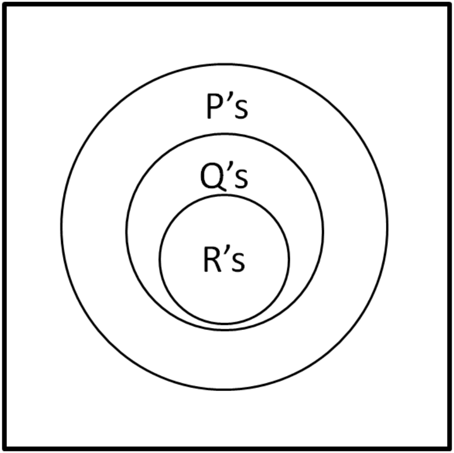

All Q's are P's.
Some P's are R's.

$\therefore$????

Verifying Quantified Arguments (Tips)

When verifying a quantified argument:

- STEP 1: Draw ”No”-quantified premises as circles.
- STEP 2: Draw ”All”-quantified premises as circles.
- STEP 3: Draw ”Some”-quantified premises as circles.
- STEP 4: Draw particular instances (e.g. Phil is a person) as points.

  (At this point, the resulting Euler Diagram satisfies all the premises.)
- STEP 5: If the resulting Euler Diagram does **not** satisfy the conclusion, then argument is **invalid**.
- Otherwise, repeat STEPS 1-5 for each case that satisfies all premises.
  - It’s best to consider cases where two or more sets are **coincident last**.
- If all cases that satify all premises also satisfy the conclusion, then argument is **valid**.

Fin

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