Graph Theory: Directed Graphs (Digraphs)
Contemporary Math
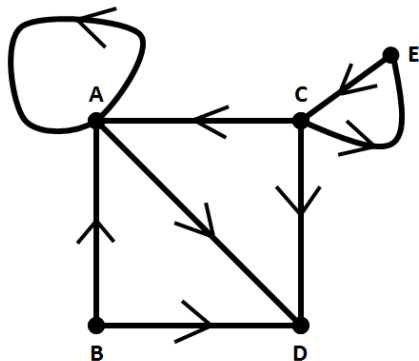
Josh Engwer

TTU

24 July 2015

Directed Graphs (Definition)

Definition

(Directed Edge, Directed Graph)

A **directed edge** (AKA an **arc**) is an edge with a direction.
A **directed graph** is a graph in which all edges are directed.

Directed Graph

Directed Paths (Definition)

Definition

(Directed Path)

A **directed path** from vertex X to vertex Y in a directed graph is a sequence of edges starting at X, following the edges in the prescribed directions, and ending at Y.

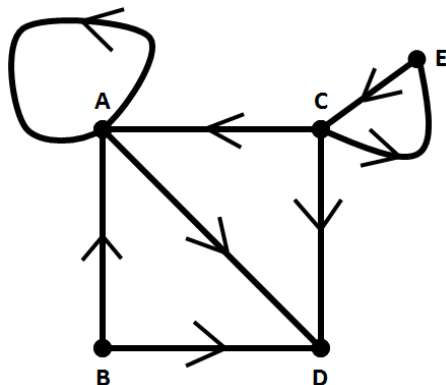The **length** of a directed path is the # of edges along that path.

The length of a **directed loop** is one (not two).

REMARK: Remember that edges cannot be repeated in a path.

Directed Paths (Example)

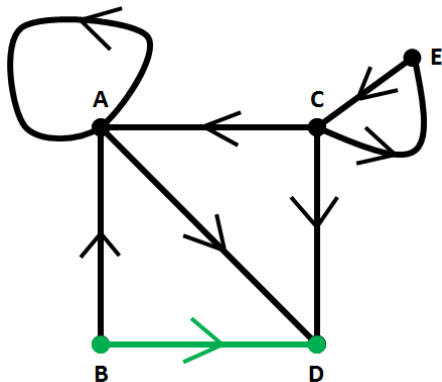**WEX 4-3-1:** Given the following digraph:

(a) Find a directed path of length 1 from B to D.

WEX 4-3-1: Given the following digraph:

(a) Find a directed path of length 1 from B to D.

Directed Path: $\boxed{BD}$

WEX 4-3-1: Given the following digraph:

(b) Find a directed path of length 2 from B to D.

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(b) Find a directed path of length 2 from B to D.

Directed Path: $\boxed{BAD}$

WEX 4-3-1: Given the following digraph:

(c) Find a directed path of length 3 from B to D.

Directed Paths (Example)

WEX 4-3-1: Given the following digraph:
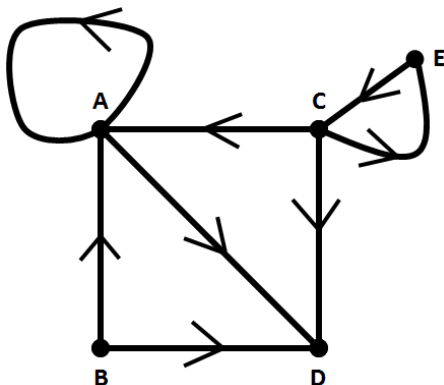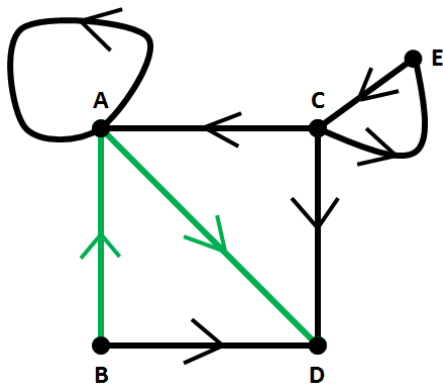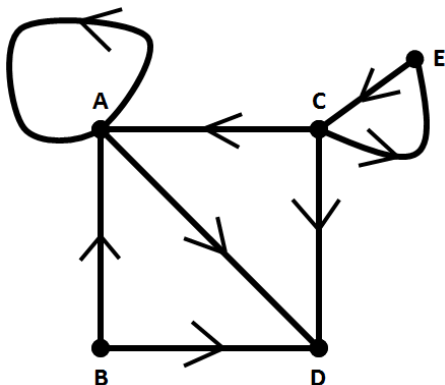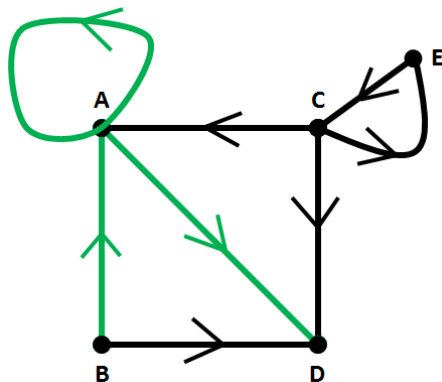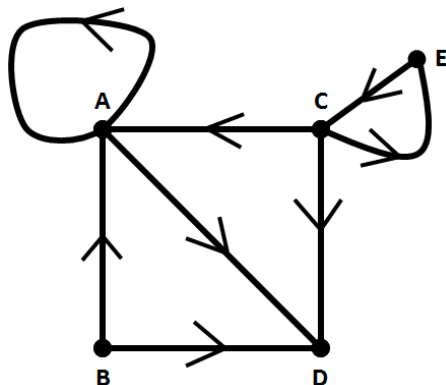
(c) Find a directed path of length 3 from B to D.

Directed Path: $\boxed{BAAD}$

WEX 4-3-1: Given the following digraph:

(d) Find a directed path of length 4 from B to D.

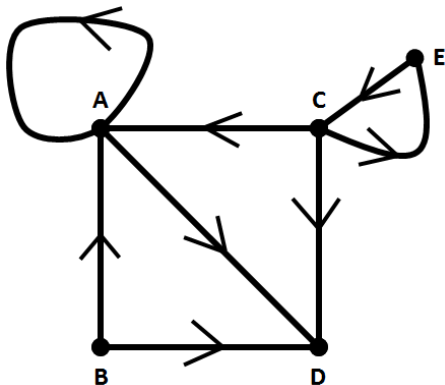**WEX 4-3-1:** Given the following digraph:

(d) Find a directed path of length 4 from B to D.
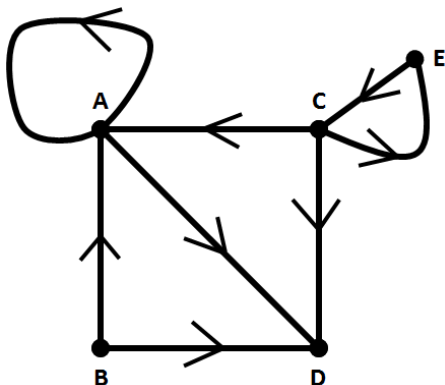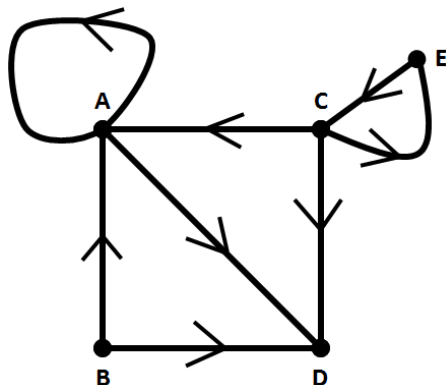

Directed Path: (Not Possible)

WEX 4-3-1: Given the following digraph:

(e) Find a directed path (of any length) from D to B.

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(e) Find a directed path (of any length) from D to B.

Directed Path: (Not Possible)

Incidence Matrix of a Digraph (Definition)

Definition

(Incidence Matrix)

An **incidence matrix** of a digraph is a rectangular array of numbers where the entries are either 0 or 1, indicating whether it's possible to traverse exactly one edge from one vertex (row) to another vertex (column).

$$\begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{ccc} A & B & C \\ \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} \end{array}$$

WEX 4-3-2: Find the corresponding incidence matrix to the digraph:

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{ccccc} A & B & C & D & E \\ \left[\begin{array}{ccccc} & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{array} \right] \end{array}$$

WEX 4-3-2: Find the corresponding incidence matrix to the digraph:

$$
\begin{array}{c}
\\ A \\ B \\ C \\ D \\ E
\end{array}
\begin{array}{ccccc}
A & B & C & D & E \\
\end{array}
\left[
\begin{array}{ccccc}
1 & 0 & 0 & 1 & 0 \\
 & & & & \\
 & & & & \\
 & & & & \\
 & & & & \\
\end{array}
\right]
$$

WEX 4-3-2: Find the corresponding incidence matrix to the digraph:

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\end{array}
\left[
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1 & 0 & 0 & 1 & 0 \\
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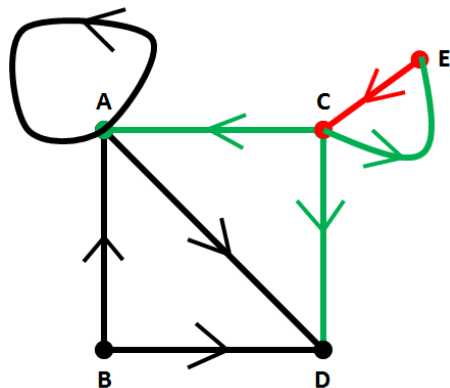 & & & & \\
 & & & & \\
 & & & &
\end{array}
\right]
$$

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\left[\begin{array}{ccccc}
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1 & 0 & 0 & 1 & 1 \\
 & & & & \\
 & & & &
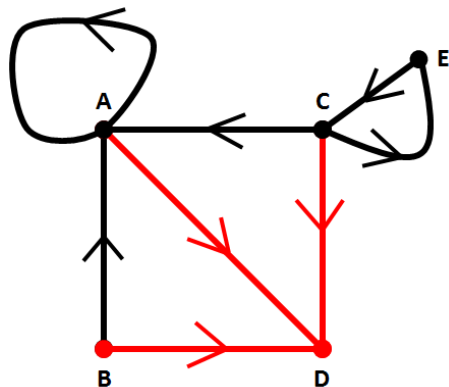\end{array}\right]
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$$

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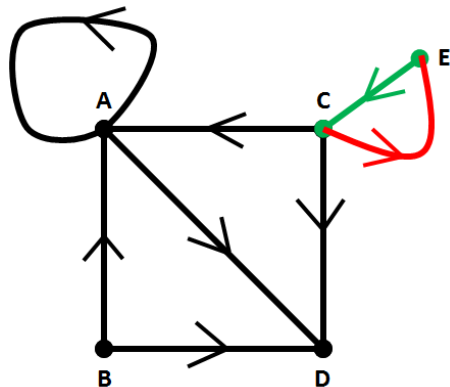0 & 0 & 0 & 0 & 0 \\
& & & &
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$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{ccccc} A & B & C & D & E \end{array}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

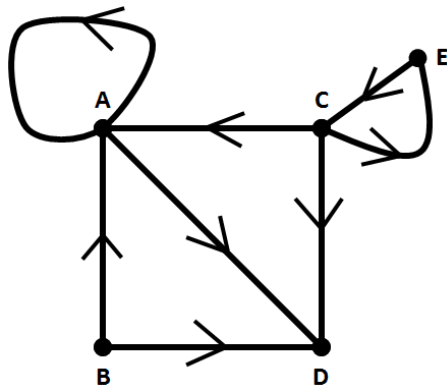**WEX 4-3-2:** Find the corresponding incidence matrix to the digraph:

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Fin.