Consumer Loans

Contemporary Math

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Installment Loans (Add-On Method)

Definition

(Installment Loan)

Installment loans are loans with a fixed number of payments. Each payment is called an **installment**.

Proposition

(Add-On Interest Method)

$$(Monthly Payment) = \frac{P+I}{n}$$

where

 $P \equiv$ Amount of Loan (Principal)

 $I \equiv Simple$ Interest due on Loan

 $n \equiv$ Number of Monthly Payments (Installments)

Credit Cards & Finance Charges

Definition

(Open-ended Credit)

Open-ended credit is a credit line that can be used up to a limit and can be paid down any time.

Examples of open-ended credit:

- Credit Cards
- Home Equity

Definition

(Finance Charge)

Credit cards incurr a **finance charge**, which is the interest charged at the end of the month.

There are two methods to compute finance charges:

- Unpaid Balance Method
- Average Daily Balance Method

Proposition

(Unpaid Balance Method)

(Finance Charge) =
$$I = Prt$$

where $P = \begin{pmatrix} Last \\ Month's \\ Balance \end{pmatrix} + \begin{pmatrix} Finance Charge \\ on Last Month's \\ Balance \end{pmatrix} + (Purchases) - (Returns) - (Payments)$ $r \equiv Annual Interest Rate$ $t = 1 month = \frac{1}{12} yr$

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May 8	Charged \$20	
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The credit card transactions during May are: The annual interest rate is 25%. Compute the finance charge using the **Unpaid Balance Method**. (May has 31 days)

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$$P = $320, r = 0.25, t = 1 \text{ month} = \frac{1}{12} \text{ year}$$

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\$320, $r = 0.25$, $t = 1$ month $= \frac{1}{12}$ year

 $\begin{pmatrix} Finance Charge on \\ Last Month's Balance \end{pmatrix} = Prt = (320)(0.25)\left(\frac{1}{12}\right) = 6.666666667$

Unpaid Balance Method (Example)

WEX 8-3-1: You begin the month of May with a credit card balance of \$320.

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$$\begin{pmatrix} Finance Charge on \\ Last Month's Balance \end{pmatrix} = Prt = (320)(0.25) \left(\frac{1}{12}\right) = 6.66666667$$

$$(Unpaid Balance) = \begin{pmatrix} Last \\ Month's \\ Balance \end{pmatrix} + \begin{pmatrix} Finance Charge \\ on Last Month's \\ Balance \end{pmatrix}$$

$$+(Purchases) - (Returns) - (Payments)$$

$$= (320) + (6.66666666667) + (20 + 15 + 10) - (10) - (40)$$

$$= $321.6666666667$$

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(Finance Charge on Last Month's Balance)

$$= Prt = (320)(0.25)\left(\frac{1}{12}\right) = 6.666666667$$

1

(Unpaid Balance) = \$321.6666666667 (Finance Charge for Next Month) =

= $(Unpaid Balance) \times r \times t$ = $(321.666666667)(0.25)(\frac{1}{12})$ = 6.701388888 $\approx [\$6.70]$

Summations are shorthand notation for a series of additions:

$$\sum_{k=1}^{4} 7 = 7 + 7 + 7 + 7$$

$$\sum_{k=1}^{5} k = 1 + 2 + 3 + 4 + 5$$

$$\sum_{k=1}^{6} k^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2$$

$$\sum_{k=1}^{7} x_k = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7$$

 Σ is the capital Greek letter *sigma*.

If space is tight, a smaller version is used: *k* is called the **index** of the summation.

$$\sum_{k=1}^{4} 7, \sum_{k=1}^{5} k, \sum_{k=1}^{6} k^2, \sum_{k=1}^{7} x_k$$

Summations & Averages

AVERAGE OF	TYPICAL FORM	SUMMATION FORM
2 numbers	$\frac{x_1 + x_2}{2}$	$\frac{1}{2}\sum_{k=1}^{2}x_{k}$
3 numbers	$\frac{x_1 + x_2 + x_3}{3}$	$\frac{1}{3}\sum_{k=1}^{3}x_k$
4 numbers	$\frac{x_1 + x_2 + x_3 + x_4}{4}$	$\frac{1}{4}\sum_{k=1}^{4} x_k$
5 numbers	$\frac{x_1 + x_2 + x_3 + x_4 + x_5}{5}$	$\frac{1}{5}\sum_{k=1}^{5}x_k$
•		
N numbers	$\frac{x_1+x_2+\cdots+x_{N-1}+x_N}{N}$	$\frac{1}{N}\sum_{k=1}^{N}x_k$

Proposition

(Average Daily Balance Method)

(Finance Charge) =
$$I = Prt$$

where

- $P = \frac{1}{N} \sum_{k=1}^{N} B_k \qquad (i.e. the average daily balance over the entire month)$
- $B_k \equiv Outstanding Balance on the kth Day of the Month$
- $N \equiv$ Number of Days in the Month
- $r \equiv$ Annual Interest Rate $t = N \text{ days } = \frac{N}{365} \text{ year(s)}$

Since the Average Daily Balance Method requires the number of days in a given month, below is a summary:

MONTH	DAYS IN THE MONTH
January	31
February	28*
March	31
April	30
May	31
June	30
July	31
August	31
September	30
October	31
November	30
December	31

* For simplicity, never assume there's a leap year

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TRANSACTION	DAY	BALANCE	# DAYS \times	BALANCE
Balance (5/1)				

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TRANSACTION	DAY	BALANCE	# DAYS \times BALANCE
Balance (5/1)	1, 2, 3, 4, 5, 6, 7	\$320	

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TRANSACTION	DAY	BALANCE	# DAYS \times BALANCE
Balance (5/1)	1, 2, 3, 4, 5, 6, 7	\$320	$7 \times \$320 = \2240

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TRANSACTION	DAY	BALANCE	# DAYS \times BALANCE
Balance (5/1)	1, 2, 3, 4, 5, 6, 7	\$320	$7 \times \$320 = \2240
Charge (5/8)			

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Charge (5/8)	8,9,10,11,12	\$320+\$20	

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Return (5/13)			

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Return (5/13)	13, 14	\$330	$2 \times \$330 = \660

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Payment (5/15)			

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Payment (5/15)	15, 16, 17, 18, 19	\$290	$5 \times \$290 = \1450

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Charge (5/20)	20, 21, 22, 23, 24, 25	\$305	$6 \times \$305 = \1830

WEX 8-3-2: You begin the month of May with a credit card balance of \$320.

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			Total = \$9770

WEX 8-3-2: You begin the month of May with a credit card balance of \$320.

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$$\implies (Avg \ Daily \ Balance) = \frac{(Total \ Daily \ Balance)}{(\# \ Days \ in \ May)} = \frac{9770}{31} = 315.1612903$$

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			Total = \$9770

 $\implies (Avg \ Daily \ Balance) = \frac{(Total \ Daily \ Balance)}{(\# \ Days \ in \ May)} = \frac{9770}{31} = 315.1612903$ $\implies (Finance \ Charge) = (Avg \ Daily \ Balance) \times (Annual \ Interest \ Rate) \times (31 \ Days)$

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Charge (5/26)	26, 27, 28, 29, 30, 31	\$315	$6 \times \$315 = \1890
			Total = \$9770

 \implies (Finance Charge) = (315.1612903)(0.25) $\left(\frac{31}{365} yr\right) = 6.6917808 \approx$ \$6.69

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Fin.