Annuities

Contemporary Math

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TTU

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Annuity (Definition)

Suppose you wish to make a very expensive purchase in the future, such as a car, a mansion, or a vacation.

Then you need to start saving for it now by setting aside a reasonable fixed amount of money regularly.

So, what should you do??

ESTABLISH AN ANNUITY!

Definition

An **(ordinary) annuity** is an interest-bearing account into which the same payment is made at the end of every compounding period.

I'll never say "ordinary annuity" – just "annuity."

- A college trust fund is an annuity.
- A child's allowance is not an annuity since allowances don't earn interest.

Sinking Fund (Definition)

Definition

A **sinking fund** is an account into which regular payments are made in order to save some specified amount in the future.

REMARK: A sinking fund is just a special type of annuity.

Typical Sinking Fund Scenarios:

- Saving for a $2000 Gaming Computer
- Saving for a $5000 Vacation Trip
- Saving for a $8000 Down Payment on a Condominium
- Saving for a $25,000 Down Payment to Start a Business
- Saving $500,000 for Retirement

Future & Present Value of an Annuity

Proposition

(Future Value of an Annuity)

$$FV = \frac{mR}{r}\left[\left(1+\frac{r}{m}\right)^{n}-1\right]$$

where

$FV \equiv$ *Future Value of the Annuity*
$R \equiv$ *Payment into the Annuity each Compounding Period*
$r \equiv$ *Annual Interest Rate*
$m \equiv$ *Number of Compounding Periods*
$t \equiv$ *Time (in years)*
$n \equiv$ *Number of Payments* $(n = mt)$

Proposition

(Present Value of an Annuity)

To find the ***present value*** *of an annuity,*
plugin all the known quantities into the above formula and solve for R*.*

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$$FV = \frac{mR}{r}\left[\left(1+\frac{r}{m}\right)^{n}-1\right] = \frac{(12)(60)}{0.06}\left[\left(1+\frac{0.06}{12}\right)^{120}-1\right]$$

$$= 12000\left[(1.005)^{120}-1\right] = 12000(0.819396734) = 9832.760808 \approx \boxed{\$9832.76}$$

Present Value of an Annuity (Example)

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$$\begin{aligned}
FV &= \frac{mR}{r}\left[\left(1+\frac{r}{m}\right)^{n}-1\right] && \leftarrow \text{(Identify relevant formula)}\\
100000 &= R\left(\frac{12}{0.06}\right)\left[\left(1+\frac{0.06}{12}\right)^{120}-1\right] && \leftarrow \text{(Plugin known quantities)}\\
100000 &= R(200)\left[(1.005)^{120}-1\right] && \leftarrow \text{(Simplify)}\\
100000 &= R(200)(0.819396734) && \leftarrow \text{(Simplify)}\\
100000 &= R(163.8793468) && \leftarrow \text{(Simplify)}\\
610.205019 &= R && \leftarrow \text{(Solve for } R\text{)}\\
\$610.21 &= R && \leftarrow \text{(Round)}
\end{aligned}$$

$$\therefore\ R = \boxed{\$610.21/\text{month}}$$

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