Amortization

Contemporary Math

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TTU

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Amortization (Definition)

Definition

Paying off a loan (plus interest) with regular equal payments is called **amortization** (AKA **financing**).

Such a loan is called an **amortized loan**.

AMORTIZED LOAN	REGULAR PAYMENTS
30-year Home Mortgage	360 equal Monthly Payments
15-year Home Mortgage	180 equal Monthly Payments
48-month Car Note	48 equal Monthly Payments

The "Dual View" of Amortization

THE SETUP: A person (consumer) borrows P dollars at r annual interest rate from a bank & agrees to pay off the loan by paying R dollars m times a year for t years ($n = mt$).

– THE TRANSACTION CAN BE VIEWED TWO WAYS –

BANKER'S POINT OF VIEW:

Account Compounded Monthly: $FV = P\left(1 + \frac{r}{m}\right)^n$

CONSUMER'S POINT OF VIEW:

Sinking Fund Compounded Monthly: $FV = \frac{mR}{r}\left[\left(1 + \frac{r}{m}\right)^n - 1\right]$

Finding Payments on an Amortized Loan

Proposition

(Finding Payments on an Amortized Loan)

$$\textit{Solve Equation} \qquad P\left(1+\frac{r}{m}\right)^{n} = \frac{mR}{r}\left[\left(1+\frac{r}{m}\right)^{n}-1\right] \qquad \textit{for } R$$

where

$P \equiv$ *Amount Borrowed (Principal)*

$R \equiv$ *Payment Amount per Period*

$r \equiv$ *Annual Interest Rate*

$m \equiv$ *Number of Periodic Payments per Year*

$t \equiv$ *Length of Time of Loan (in* ***years****)*

$n \equiv$ *Number of Payments* $(n = mt)$

Finding Present Value of an Annuity via Amortization

Proposition

(Finding the Present Value of an Annuity)

$$\textit{Solve Equation} \qquad P\left(1+\frac{r}{m}\right)^{n} = \frac{mR}{r}\left[\left(1+\frac{r}{m}\right)^{n}-1\right] \qquad \textit{for } P$$

where

$P \equiv$ *Amount Borrowed (Principal)*

$R \equiv$ *Payment Amount per Period*

$r \equiv$ *Annual Interest Rate*

$m \equiv$ *Number of Periodic Payments per Year*

$t \equiv$ *Length of Time of Loan (in* ***years****)*

$n \equiv$ *Number of Payments* $(n = mt)$

Amortization Schedule (Definition)

Payments on an amortized loan partly pay off the principal & partly pay interest on the outstanding principal.
Over time, each successive payment pays more toward principal & less toward interest.

Definition

An **amortization schedule** is a list showing payment-by-payment how much is going towards the principal & interest.

The key formulas for building an amortization schedule are:

- $(\textit{Monthly Interest Rate}) = \frac{1}{12} \times (\textit{Annual Interest Rate})$

  For each month:
- $(\textit{Interest Paid}) = (\textit{Last Balance}) \times (\textit{Monthly Interest Rate}) \times (1\ \textit{Month})$
- $(\textit{Monthly Payment}) - (\textit{Interest Paid}) = (\textit{Paid on Principal})$
- $(\textit{New Balance}) = (\textit{Last Balance}) - (\textit{Paid on Principal})$

Amortization Schedule (Example)

$10,000 Loan at 18% Annual Interest for 4 Years

	PAYMENT NUMBER	MONTHLY PAYMENT	INTEREST PAID	PAID ON PRINCIPAL	REMAINING BALANCE
Initial					$10,000.00
Month 1	1	$293.75	$150.00	$143.75	$9856.25
Month 2	2	$293.75	$147.84	$145.91	$9710.34
Month 3	3	$293.75	$145.66	$148.09	$9562.25
⋮	⋮	⋮	⋮	⋮	⋮
End Year 1	12	$293.75	$124.42	$169.33	$8125.33
End Year 2	24	$293.75	$91.30	$202.45	$5883.93
End Year 3	36	$293.75	$51.69	$242.06	$3204.08
End Year 4	48	$293.75	$4.34	$289.41	$0.00

$$(Interest\ Paid) = (Last\ Balance) \times (Monthly\ Interest\ Rate) \times (1\ Month)$$
$$(Monthly\ Payment) - (Interest\ Paid) = (Paid\ on\ Principal)$$
$$(New\ Balance) = (Last\ Balance) - (Paid\ on\ Principal)$$

Refinancing a Loan

Suppose you have to borrow money (for a house, say) at a high interest rate. Moreover, you notice a year later that the interest rates decline markedly. So now, you want to pay off the remaining debt on the loan by taking out a second loan at the lower interest rate.
This is called **refinancing** the loan.

Definition

Refinancing a loan is the process of taking out a second loan to pay off the first loan, but at a lower interest rate.

Why Refinance??

- Refinancing lowers the size of the monthly payments & reduces the total interest paid.

Refinancing a Loan (Why it Matters)

MONTHLY PAYMENTS ON A $1000.00 LOAN

INTEREST RATE	3 YEARS	10 YEARS	20 YEARS	30 YEARS
4% annual	$29.53	$10.12	$6.06	$4.77
6% annual	$30.42	$11.10	$7.16	$6.00
8% annual	$31.34	$12.13	$8.36	$7.34
10% annual	$32.27	$13.22	$9.65	$8.78
12% annual	$33.21	$14.35	$11.01	$10.29

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Savings when refinancing from 6% to 4% for **3 years**:

$$(3 \text{ yrs})(12 \text{ months/yr})\Big[(\$30.42/\text{month}) - (\$29.53/\text{month})\Big] = \mathbf{\$32.04}$$

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Savings when refinancing from 12% to 4% for **3 years**:

$$(3 \text{ yrs})(12 \text{ months/yr})\Big[(\$33.21/\text{month}) - (\$29.53/\text{month})\Big] = \mathbf{\$132.48}$$

Refinancing a Loan (Why it Matters)

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Savings when refinancing from 12% to 4% for **30 years**:

$$(30 \text{ yrs})(12 \text{ months/yr})\Big[(\$10.29/\text{month}) - (\$4.77/\text{month})\Big] = \textbf{\$1987.20} \quad !!!$$

So, you end up saving **almost double the loan amount**!!

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Savings when refinancing from 6% to 4% for **30 years**:

$$(30 \text{ yrs})(12 \text{ months/yr})\Big[(\$6.00/\text{month}) - (\$4.77/\text{month})\Big] = \textbf{\$442.80} \quad !!!$$

So, you end up saving **almost half the loan amount**!!
So, reducing the interest rate by a little can result in big savings long-term!

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