Annual Percentage Rate (APR)
Contemporary Math

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TTU

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Annual Percentage Rate (Definition)

Suppose the interest rate is not given or an **add-on interest rate** is given.

If you use the **Add-On Interest Method** (§8.3), the "true" interest rate **changes every payment period**!!!

– SO WHAT IS THE "TRUE" INTEREST RATE??? –

Definition

The **annual percentage rate (APR)** is a standardized version of the "true" interest rate on a loan.

The APR can be computed using the **simple interest formula** (§8.2) and solving for the interest rate (r), but the simple interest occurs every payment!

So, if the loan is to be payed off once a month for 5 years, then the resulting equation involves $5 \times 12 = 60$ terms!!!

What follows are two simple methods to compute/estimate the APR.

Computing the APR for Loans (Procedure)

TABLE OF FINANCE CHARGE PER $100   (FCPH):

NUMBER OF PAYMENTS	10%	11%	12%	APR 13%	14%	15%	16%
6	$2.94	$3.23	$3.53	$3.83	$4.12	$4.42	$4.72
12	$5.50	$6.06	$6.62	$7.18	$7.74	$8.31	$8.88
24	$10.75	$11.86	$12.98	$14.10	$15.23	$16.37	$17.51
36	$16.16	$17.86	$19.57	$21.30	$23.04	$24.80	$26.57
48	$21.74	$24.06	$26.40	$28.77	$31.17	$33.59	$36.03

Proposition

(Computing the APR for Loans)

STEP 0: $P \equiv$ Amount Borrowed, $FC \equiv$ Finance Charge, $n \equiv$ # Payments

STEP 1: Compute **FCPH** $= \dfrac{(\text{Finance Charge})}{(\text{Amount Borrowed})} \times 100 = \dfrac{FC}{P} \times 100$

*STEP 2: Find the **closest entry** in Row[n] of above table to **FCPH***

*STEP 3: The **column heading of the table entry** is the APR*

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WEX 8-6-1:
You pay off a $4000.00 loan by making a payment every 4 months for 2 years.
If you pay a finance charge of $180.00, what is the APR?

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$$\text{STEP 0: } P = \$4000, \ FC = \$180$$

$$n = \frac{1 \text{ payment}}{4 \text{ months}} \times \frac{12 \text{ months}}{1 \text{ yr}} \times 2 \text{ yrs} = 6 \text{ payments}$$

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$$\text{STEP 2: In Row}[n] = \text{Row}[6] \text{ (in blue), find closest entry to } FCPH = \$4.50$$

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STEP 2: In Row[6], closest entry to $FCPH$ is $4.42 (in green)

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$$\text{STEP 2: In Row[6], closest entry to } FCPH \text{ is } \$4.42 \text{ (in green)}$$
$$\text{STEP 3: The APR is the column heading for the entry, } 15\% \text{ (in red)}$$

Estimating the APR for Add-on Interest Loans

Without a table or technology, it would be difficult to calculate the APR.

Fortunately for **add-on interest loans**, there's a formula to **estimate** the APR:

Proposition

(Approximation of APR for Add-on Interest Loans)

$$APR \approx \frac{2nr}{n+1}$$

where

r ≡ Annual Interest Rate
n ≡ Number of Payments

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