Texas Tech University
MATH1321 -- Trigonometry

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11/24/2010

Evaluating Some Trig. Values Two Ways: Same Answer, Different Forms

When finding exact values of trig functions of angles, one has to choose the correct identity.

For example, to compute cos 22.5°, you have to use the Half-Angle Identity for cosine.

However, for some angles, you have the choice of either using a Sum/Difference Identity or a Half-Angle Identity.

In such a case, finding the answer using two different identities can yield stark differences in the answer in form, but the actual decimal answer will be the same.

Let's compute sin15° as an example.

Evaluating sin15° via the Difference Identity:

$$\sin 15° = \sin(45° - 30°) = \sin 45° \cos 30° - \cos 45° \sin 30° = \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \times \frac{1}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

Evaluating sin15° via the Half-Angle Identity:

$$\sin 15° = \sin\left(\frac{1}{2} \times 30°\right) = \pm\sqrt{\frac{1 - \cos 30°}{2}} = \frac{\sqrt{2 - \sqrt{3}}}{2}$$

Notice, that the forms of the two answers are markedly distinct!

But, does $\frac{\sqrt{2-\sqrt{3}}}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}$??

Well, one way is to find the decimal approximations to each expression:

$$\frac{\sqrt{2-\sqrt{3}}}{2} \approx 0.25882 \quad \text{and} \quad \frac{\sqrt{6} - \sqrt{2}}{4} \approx 0.25882$$

Another way is to verify their equality algebraically:

$$\frac{2 - \sqrt{3}}{2} = \frac{\sqrt{6} - \sqrt{2}}{4} \Rightarrow 2\sqrt{2 - \sqrt{3}} = \sqrt{6} - \sqrt{2}$$

$$\Rightarrow 4(2 - \sqrt{3}) = 6 - 2\sqrt{12} + 2 \Rightarrow 8 - 4\sqrt{3} = 8 - 2\sqrt{12}$$

$$\Rightarrow 4 - 2\sqrt{3} = 4 - \sqrt{12}$$

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$$\Rightarrow 0 = 0$$

Since the equation '0 = 0' is a true statement, that implies these two different forms do indeed have the same value.