Texas Tech University
MATH1321 -- Trigonometry

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Quick Method of graphing secant or cosecant functions, regardless of complexity

The trick that the textbook [1] uses (plotting the respective sine/cosine graph first) has a few downsides:

- It works best only when the function is very 'simple' (e.g. $y = 3\sec(x)$)
- It requires graphing a second function, which takes more time and clutters the graph
- It requires knowledge of amplitude, horizontal stretch, phase shift, vertical shift, and mirror image.
- If the sine/cosine function is graphed wrong, then the secant/cosecant graph will be wrong automatically!

In fact, the textbook [1] does not even have an example of graphing a very complicated secant/cosecant function!

The method presented here mirrors that used in Section 4.2 (known as 'Method 1' in that section). In fact, 'Method 1' is used exclusively for more complicated sine/cosine graphs versus using shifts & stretches (see Example 5, page 162). The main difference here is that we must consider where the vertical asymptotes occur.

This method is also used in Section 4.3 (graphing tangent/cotangent functions) – see Example 5, page 173.
The main difference is the first part of STEP 1:
the argument of tangent is set between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$,
and the argument of cotangent is set between 0 and π .

These differences are needed to ensure that an entire period of tangent/cotangent is plotted as well as ensure that a pair of vertical asymptotes are included.

Note, that the examples presented below are in the most complicated form that secant/cosecant can be:

$y = c + a \sec(bx - d)$ or $y = c + a \csc(bx - d)$

but the method used here works for simpler secant/cosecant functions (i.e. $c = 0$, $d = 0$, $a = 1$, and/or $b = 1$)

The last page has some harder problems to try to help reinforce the technique.

Example (E1): Sketch $y = -\frac{1}{2} - \frac{1}{4}\sec(4x - \pi)$

STEP 1: Find the 'key values' of *x*.

Set the argument of sec(4*x* - π) between 0 and 2π :

$$0 \leqslant 4x - \pi \leqslant 2\pi \Rightarrow \pi \leqslant 4x \leqslant 3\pi \Rightarrow \frac{\pi}{4} \leqslant x \leqslant \frac{3\pi}{4} \Rightarrow x \in \left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$$

Partition the interval into 4 pieces. In other words, divide the length of the interval by 4 :

$$\text{Length of interval } \left[\frac{\pi}{4}, \frac{3\pi}{4}\right] \div 4 = \left(\frac{3\pi}{4} - \frac{\pi}{4}\right) \div 4 = \frac{\pi}{2} \div 4 = \frac{\pi}{2}\left(\frac{1}{4}\right) = \frac{\pi}{8}$$

Now, start with the lower end of the interval (which is $\frac{\pi}{4}$), and add $\frac{\pi}{8}$ to it repetitively until the upper end of the interval is reached $\left(\frac{3\pi}{4}\right)$. There should be only five values total!

The key values of *x* are: $\frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}, \frac{5\pi}{8}, \frac{3\pi}{4}$

STEP 2: Build a table with the key values of *x* and evaluate the function at each key value:

x	$y = -\frac{1}{2} - \frac{1}{4}\sec(4x - \pi)$
$\frac{\pi}{4}$	$-\frac{1}{2} - \frac{1}{4}\sec(\mathbf{0}) = -\frac{3}{4}$
$\frac{3\pi}{8}$	$-\frac{1}{2} - \frac{1}{4}\sec\left(\boldsymbol{\frac{\pi}{2}}\right) = \textit{UNDEFINED}$
$\frac{\pi}{2}$	$-\frac{1}{2} - \frac{1}{4}\sec(\boldsymbol{\pi}) = -\frac{1}{4}$
$\frac{5\pi}{8}$	$-\frac{1}{2} - \frac{1}{4}\sec\left(\boldsymbol{\frac{3\pi}{2}}\right) = \textit{UNDEFINED}$
$\frac{3\pi}{4}$	$-\frac{1}{2} - \frac{1}{4}\sec(\boldsymbol{2\pi}) = -\frac{3}{4}$

* Notice that the arguments of secant reduce to quadrantal angles (in bold above) -- that's no accident!

STEP 3: Plot the asymptotes from table (where y is ***UNDEFINED***) with dashed vertical lines at: $x = \frac{3\pi}{8}$, $x = \frac{5\pi}{8}$

STEP 4: Plot the remaining points from table: $\left(\frac{\pi}{4}, -\frac{3}{4}\right), \left(\frac{\pi}{2}, -\frac{1}{4}\right), \left(\frac{3\pi}{4}, -\frac{3}{4}\right)$

STEP 5: Draw an '**upward parabola**' at each point in STEP 4 with a **larger** ***y*-value**: $\left(\frac{\pi}{2}, -\frac{1}{4}\right)$

Draw a '**downward parabola**' at each point in STEP 4 with a **smaller** ***y*-value**: $\left(\frac{\pi}{4}, -\frac{3}{4}\right), \left(\frac{3\pi}{4}, -\frac{3}{4}\right)$

STEP 6: At this point of the sketch, one entire period of the function has been graphed, therefore one can quickly extend/replicate the graph at other intervals of *x* as needed, using what has already been sketched.

NOTE: Make sure each point in STEP 4 is halfway between (horizontally) the two asymptotes surrounding it!

FINAL STEPS: Label axes and make sure key points in STEP 4 are either explicitly labeled or inferred from axes.

<u>Example (E2):</u> Sketch $y = 5 + 3\csc\left(\frac{1}{2}x + \frac{\pi}{2}\right)$

STEP 1: Find the 'key values' of *x*.

Set the <u>argument</u> of $\csc\left(\frac{1}{2}x + \frac{\pi}{2}\right)$ between 0 and 2π :

$$0 \leqslant \frac{1}{2}x + \frac{\pi}{2} \leqslant 2\pi \Rightarrow -\frac{\pi}{2} \leqslant \frac{1}{2}x \leqslant \frac{3\pi}{2} \Rightarrow -\pi \leqslant x \leqslant 3\pi \Rightarrow x \in [-\pi, 3\pi]$$

Partition the interval into 4 pieces. In other words, divide the length of the interval by 4 :

Length of interval $[-\pi, 3\pi] \div 4 = (3\pi - (-\pi)) \div 4 = 4\pi \div 4 = \pi$

Now, start with the lower end of the interval (which is $-\pi$), and add π to it repetitively until the upper end of the interval is reached (3π). There should be only five values total!

The key values of *x* are: $-\pi$, 0 , π , 2π , 3π

STEP 2: Build a table with the key values of *x* and evaluate the function at each key value:

x	$y = 5 + 3\csc\left(\frac{1}{2}x + \frac{\pi}{2}\right)$
$-\pi$	$5 + 3\csc(\mathbf{0}) = UNDEFINED$
0	$5 + 3\csc\left(\boldsymbol{\frac{\pi}{2}}\right) = 8$
π	$5 + 3\csc(\boldsymbol{\pi}) = UNDEFINED$
2π	$5 + 3\csc\left(\boldsymbol{\frac{3\pi}{2}}\right) = 2$
3π	$5 + 3\csc(\mathbf{2}\boldsymbol{\pi}) = UNDEFINED$

* Notice that the arguments of cosecant reduce to quadrantal angles (in bold above) -- that's no accident!

STEP 3: Plot the asymptotes from table (where y is ***UNDEFINED***) with dashed vertical lines at:
$x = -\pi$, $x = \pi$, $x = 3\pi$

STEP 4: Plot the remaining points from table: $(0,8), (2\pi, 2)$

STEP 5: Draw an '**upward parabola**' at each point in STEP 4 with a **larger *y*-value**: $(0,8)$

Draw a '**downward parabola**' at each point in STEP 4 with a **smaller *y*-value**: $(2\pi, 2)$

STEP 6: At this point of the sketch, one entire period of the function has been graphed, therefore one can quickly extend/replicate the graph at other intervals of *x* as needed, using what has already been sketched.

NOTE: Make sure each point in STEP 4 is halfway between (horizontally) the two asymptotes surrounding it!
FINAL STEPS: Label axes and make sure key points in STEP 4 are either explicitly labeled or inferred from axes.

Practice Problems: For each problem, sketch the graph over a two-period interval.

(P1) $y = 2 - 7\sec\left(x - \frac{\pi}{5}\right)$

(P2) $w = -\frac{5}{6} + \frac{2}{3}\csc(11t + \pi)$

(P3) $y = \sqrt{13} - \sqrt{13}\csc\left(2x - \frac{2\pi}{3}\right)$

(P4) $z = \frac{2}{\pi^3} - \frac{5}{\pi^3}\sec(\pi x - \pi)$

NOTE: Problems like (P3) & (P4) are not in the textbook [1], therefore such problems will not be on an exam. Problem (P2) is complex enough algebraically that a similar problem may be a possible bonus question on an exam.

References

[1] M. L. Lial, J. E. Hornsby, D. Schneider. *Trigonometry.* Pearson, Boston, MA, 9th Edition, 2009.