Texas Tech University
MATH1321 -- Trigonometry

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How to painlessly solve SSA triangles

Here's a concise summary of the process that the textbook [1] lays out to solving the ambiguous SSA layout:

1. Use Law of Sines to find the **unknown angle opposite the known side**.
   Here's a useful mnemonic device: MADES = ' Mystery Angle Displaced from Established Side'

If upon using the $[\sin^{-1}]$ calculator button you get a **DOMAIN ERROR** then NO TRIANGLE EXISTS -- DONE!

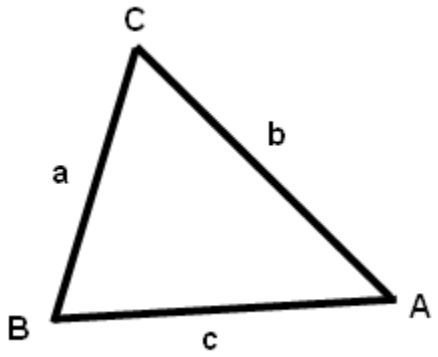
Else, solve the triangle: Use Law of Sines to find the unknown side & the Angle Sum of Triangles to find last angle.

2. Now, find the supplement of the unknown angle opposite the known side.

If MADES + (the given angle) $\geq 180°$, then NO 2ND TRIANGLE EXISTS -- DONE!

Else, solve 2nd triangle: Use Law of Sines to find the unknown side & the Angle Sum of Triangles to find last angle.

Triangle Labeling Convention used by the textbook [1]:



Angle Sum of Triangles: $A + B + C = 180°$

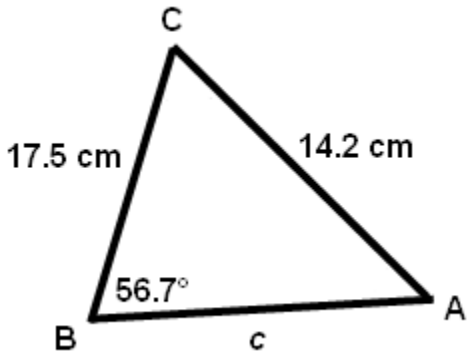
Law of Sines: $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

Example (E5): Solve triangle ABC with B = 56.7°, a = 17.5 cm, b = 14.2 cm

STEP 0: MAKE SURE CALCULATOR IS SET TO **DEGREE** MODE!

STEP 1: Sketch triangle ABC (not to scale) and determine layout

Upon sketching triangle ABC and labeling what is given using the standard convention in the textbook [1], one realizes that this is an **SSA** case, implying that the **Law of Sines** must be invoked.



PART I: Solve 1st triangle (if it exists)

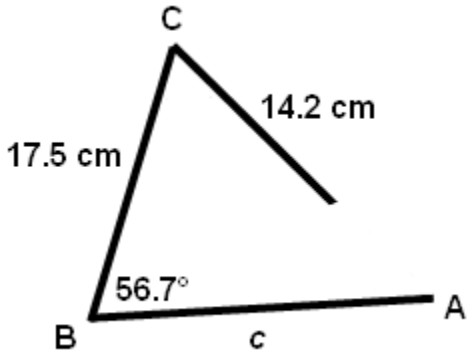
STEP 2: Find the unknown angle opposite a known side, in this case angle A:
[Mnemonic Device : MADES = 'Mystery Angle Displaced from Established Side']

$$\frac{\sin A}{a} = \frac{\sin B}{b} \Rightarrow \frac{\sin A}{17.5\,\text{cm}} = \frac{\sin 56.7^\circ}{14.2\,\text{cm}} \Rightarrow \sin A = 1.030044283 \Rightarrow \textbf{DOMAIN ERROR}$$

Since 1.030044283 falls outside the range of sine $[-1,1]$, angle A is undefined.

Therefore, NO TRIANGLE EXISTS -- DONE!

Drawing the triangle carefully to scale reveals why triangle ABC does not really exist:

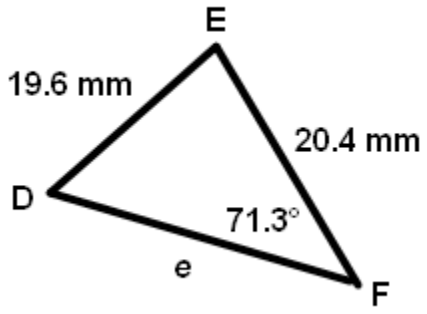


Example (E6): Solve triangle DEF with $F = 71.3°$, $d = 20.4$ mm, $f = 19.6$ mm

STEP 0: MAKE SURE CALCULATOR IS SET TO **DEGREE** MODE!

STEP 1: Sketch triangle DEF (not to scale) and determine layout

Upon sketching triangle DEF and labeling what is given using the standard convention in the textbook [1], one realizes that this is an **SSA** case, implying that the **Law of Sines** must be invoked.



PART I: Solve 1st triangle (if it exists)

STEP 2: Find the unknown angle opposite a known side, in this case angle D:
[Mnemonic Device : MADES = 'Mystery Angle Displaced from Established Side']

$$\frac{\sin D}{d} = \frac{\sin F}{f} \Rightarrow \frac{\sin D}{20.4\,\text{mm}} = \frac{\sin 71.3°}{19.6\,\text{mm}} \Rightarrow \sin D = 0.9858719217 \Rightarrow D = 80.35745181° \Rightarrow \mathbf{D = 80.4°}$$

STEP 3: Using the Angle Sum of Triangles, find the last angle (in this case angle E):

$$D + E + F = 180° \Rightarrow 80.4° + E + 71.3° = 180° \Rightarrow \mathbf{E = 28.3°}$$

STEP 4: Find the unknown side (in this case, side *e*):

$$\frac{\sin F}{f} = \frac{\sin E}{e} \Rightarrow \frac{\sin 71.3°}{19.6\,\text{mm}} = \frac{\sin 28.3°}{e} \Rightarrow e = 9.809995854\,\text{mm} \Rightarrow \mathbf{e = 9.81\ mm}$$

PART II: Solve 2nd triangle (if it exists)

STEP 5: Find the supplement of MADES, i.e. supplement = $180° - D = 180° - 80.35745181° = 99.64254819°$
Let MADES be this new value, i.e. $D = 99.64254819° \Rightarrow \mathbf{D = 99.6°}$

STEP 6: Is MADES + (given angle) $\geq 180°$? $D + F = 99.6° + 71.3° = 170.9° < 180° \Rightarrow$ The answer is NO
Since the answer is NO, a second triangle exists.

STEP 7: Using the Angle Sum of Triangles, find the last angle (in this case angle E):

$$D + E + F = 180° \Rightarrow 99.6° + E + 71.3° = 180° \Rightarrow \mathbf{E = 9.10°}$$

STEP 8: Find the unknown side (in this case, side *e*):

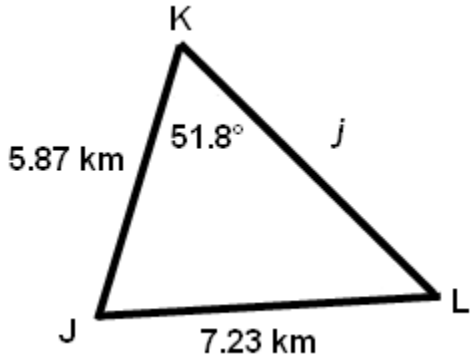
$$\frac{\sin F}{f} = \frac{\sin E}{e} \Rightarrow \frac{\sin 71.3°}{19.6\,\text{mm}} = \frac{\sin 9.10°}{e} \Rightarrow e = 3.272660982\,\text{mm} \Rightarrow \mathbf{e = 3.27\ mm}$$

Example (E7): Solve triangle JKL with $K = 71.3°$, $k = 7.23$ km, $l = 5.87$ km

STEP 0: MAKE SURE CALCULATOR IS SET TO **DEGREE** MODE!

STEP 1: Sketch triangle JKL (not to scale) and determine layout

Upon sketching triangle JKL and labeling what is given using the standard convention in the textbook [1], one realizes that this is an **SSA** case, implying that the **Law of Sines** must be invoked.



PART I: Solve 1st triangle (if it exists)

STEP 2: Find the unknown angle opposite a known side, in this case angle L:
[Mnemonic Device : MADES = 'Mystery Angle Displaced from Established Side']

$$\frac{\sin L}{l} = \frac{\sin K}{k} \Rightarrow \frac{\sin L}{5.87\,\text{km}} = \frac{\sin 51.8°}{7.23\,\text{km}} \Rightarrow \sin L = 0.6380331899 \Rightarrow L = 39.64531528° \Rightarrow \mathbf{L = 39.6°}$$

STEP 3: Using the Angle Sum of Triangles, find the last angle (in this case angle J):

$$J + K + L = 180° \Rightarrow J + 51.8° + 39.6° = 180° \Rightarrow \mathbf{J = 88.6°}$$

STEP 4: Find the unknown side (in this case, side *e*):

$$\frac{\sin J}{j} = \frac{\sin K}{k} \Rightarrow \frac{\sin 88.6°}{j} = \frac{\sin 51.8°}{7.23\,\text{km}} \Rightarrow j = 9.197402013\,\text{km} \Rightarrow \mathbf{j = 9.20\ km}$$

PART II: Solve 2nd triangle (if it exists)

STEP 5: Find the supplement of MADES, i.e. supplement = $180° - L = 180° - 39.64531528° = 140.3546847°$
Let MADES be this new value, i.e. $L = 140.3546847° \Rightarrow \mathbf{L = 140°}$

STEP 6: Is MADES + (given angle) $\geq 180°$? $K + L = 51.8° + 140° = 191.8° > 180° \Rightarrow$ The answer is YES
Since the answer is YES, second triangle DOES NOT EXIST -- DONE!

References

[1] M. L. Lial, J. E. Hornsby, D. Schneider. *Trigonometry*. Pearson, Boston, MA, 9th Edition, 2009.