Texas Tech University
MATH1321 -- Trigonometry

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Quick Method of graphing tangent or cotangent functions, regardless of complexity

The textbook [1] does not have an example of graphing a very complicated tangent/cotangent function!

The method presented here mirrors the Guidelines mentioned in Section 4.3 (page 171).

This method is also used in Example 5, page 173.

The main difference from the method used for secant/cosecant functions is the first part of STEP 1:
the argument of tangent is set between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$,
and the argument of cotangent is set between 0 and π .

These differences are needed to ensure that an entire period of tangent/cotangent is plotted as well as ensure that a pair of vertical asymptotes are included.

Note, that the examples presented below are in the most complicated form that tangent/cotangent can be:

$y = c + a \tan(bx - d)$ or $y = c + a \cot(bx - d)$

but the method also works fine for simpler tangent/cotangent functions (i.e. $c = 0$, $d = 0$, $a = 1$, and/or $b = 1$)

The last page has some harder problems to try to help reinforce the technique.

Example (E3): Sketch $y = \frac{8}{9} - \frac{2}{3}\tan\left(4x - \pi\right)$

STEP 1: Find the 'key values' of x.

Set the argument of $\tan(4x - \pi)$ between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$:

$$-\frac{\pi}{2} \leqslant 4x - \pi \leqslant \frac{\pi}{2} \Rightarrow \frac{\pi}{2} \leqslant 4x \leqslant \frac{3\pi}{2} \Rightarrow \frac{\pi}{8} \leqslant x \leqslant \frac{3\pi}{8} \Rightarrow x \in \left[\frac{\pi}{8}, \frac{3\pi}{8}\right]$$

Partition the interval into 4 pieces. In other words, divide the length of the interval by 4 :

$$\text{Length of interval } \left[\frac{\pi}{8}, \frac{3\pi}{8}\right] \div 4 = \left(\frac{3\pi}{8} - \frac{\pi}{8}\right) \div 4 = \frac{\pi}{4} \div 4 = \frac{\pi}{4}\left(\frac{1}{4}\right) = \frac{\pi}{16}$$

Now, start with the lower end of the interval (which is $\frac{\pi}{8}$), and add $\frac{\pi}{16}$ to it repetitively until the upper end of the interval is reached ($\frac{3\pi}{8}$). There should be only five values total!

The key values of x are: $\frac{\pi}{8}, \frac{3\pi}{16}, \frac{\pi}{4}, \frac{5\pi}{16}, \frac{3\pi}{8}$

STEP 2: Build a table with the key values of x and evaluate the function at each key value:

x	$y = \frac{8}{9} - \frac{2}{3}\tan\left(4x - \pi\right)$
$\frac{\pi}{8}$	$\frac{8}{9} - \frac{2}{3}\tan\left(\boldsymbol{-\frac{\pi}{2}}\right) = UNDEFINED$
$\frac{3\pi}{16}$	$\frac{8}{9} - \frac{2}{3}\tan\left(\boldsymbol{-\frac{\pi}{4}}\right) = \frac{14}{9}$
$\frac{\pi}{4}$	$\frac{8}{9} - \frac{2}{3}\tan\left(\boldsymbol{0}\right) = \frac{8}{9}$
$\frac{5\pi}{16}$	$\frac{8}{9} - \frac{2}{3}\tan\left(\boldsymbol{\frac{\pi}{4}}\right) = \frac{2}{9}$
$\frac{3\pi}{8}$	$\frac{8}{9} - \frac{2}{3}\tan\left(\boldsymbol{\frac{\pi}{2}}\right) = UNDEFINED$

* Notice: the arguments of tangent reduce to either 0, $\pm\frac{\pi}{2}$ or $\pm\frac{\pi}{4}$ (in bold above) -- that's no accident!

STEP 3: Plot the asymptotes from table (where y is ***UNDEFINED***) with dashed vertical lines at: $x = \frac{\pi}{8}$, $x = \frac{3\pi}{8}$

STEP 4: Plot the remaining points from table: $\left(\frac{3\pi}{16}, \frac{14}{9}\right), \left(\frac{\pi}{4}, \frac{8}{9}\right), \left(\frac{5\pi}{16}, \frac{2}{9}\right)$

STEP 5: Connect the points with a smooth curve, approaching the vertical asymptotes.

STEP 6: At this point of the sketch, one entire period of the function has been graphed, therefore one can quickly extend/replicate the graph at other intervals of x as needed, using what has already been sketched.

FINAL STEPS: Label axes and make sure key points in STEP 4 are either explicitly labeled or inferred from axes.

Example (E4): Sketch $z = -6 + 3\cot\left(\frac{1}{3}t - \frac{\pi}{3}\right)$

STEP 1: Find the 'key values' of t.

Set the argument of $\cot\left(\frac{1}{3}t - \frac{\pi}{3}\right)$ between 0 and π :

$$0 \leqslant \frac{1}{3}t - \frac{\pi}{3} \leqslant \pi \Rightarrow \frac{\pi}{3} \leqslant \frac{1}{3}t \leqslant \frac{4\pi}{3} \Rightarrow \pi \leqslant t \leqslant 4\pi \Rightarrow t \in [\pi, 4\pi]$$

Partition the interval into 4 pieces. In other words, divide the length of the interval by 4 :

$$\text{Length of interval } [\pi, 4\pi] \div 4 = (4\pi - \pi) \div 4 = 3\pi \div 4 = \frac{3\pi}{4}$$

Now, start with the lower end of the interval (which is π), and add $\frac{3\pi}{4}$ to it repetitively until the upper end of the interval is reached (4π) . There should be only five values total!

The key values of t are: π , $\frac{7\pi}{4}$, $\frac{5\pi}{2}$, $\frac{13\pi}{4}$, 4π

STEP 2: Build a table with the key values of t and evaluate the function at each key value:

t	$z = -6 + 3\cot\left(\frac{1}{3}t - \frac{\pi}{3}\right)$
π	$-6 + 3\cot(\mathbf{0}) = \mathit{UNDEFINED}$
$\frac{7\pi}{4}$	$-6 + 3\cot\left(\boldsymbol{\frac{\pi}{4}}\right) = -3$
$\frac{5\pi}{2}$	$-6 + 3\cot\left(\boldsymbol{\frac{\pi}{2}}\right) = -6$
$\frac{13\pi}{4}$	$-6 + 3\cot\left(\boldsymbol{\frac{3\pi}{4}}\right) = -9$
4π	$-6 + 3\cot(\boldsymbol{\pi}) = \mathit{UNDEFINED}$

* Notice: the arguments of co-tangent reduce to quadrantal angles, $\frac{\pi}{4}$, or $\frac{3\pi}{4}$ (in bold above) -- that's no accident!

STEP 3: Plot the asymptotes from table (where z is ***UNDEFINED***) with dashed vertical lines at: $t = \pi$, $t = 4\pi$

STEP 4: Plot the remaining points from table: $\left(\frac{7\pi}{4}, -3\right), \left(\frac{5\pi}{2}, -6\right), \left(\frac{13\pi}{4}, -9\right)$

STEP 5: Connect the points with a smooth curve, approaching the vertical asymptotes.

STEP 6: At this point of the sketch, one entire period of the function has been graphed, therefore one can quickly extend/replicate the graph at other intervals of t as needed, using what has already been sketched.

FINAL STEPS: Label axes and make sure key points in STEP 4 are either explicitly labeled or inferred from axes.

Practice Problems: For each problem, sketch the graph over a two-period interval.

(P5) $y = 2 - 7\cot\left(x - \dfrac{\pi}{5}\right)$

(P6) $w = -\dfrac{5}{6} + \dfrac{2}{3}\tan(11\beta + \pi)$

(P7) $y = \sqrt{7} + \sqrt{7}\tan\left(2x - \dfrac{2\pi}{3}\right)$

(P8) $q = \dfrac{2}{\pi^3} + \dfrac{3}{\pi^3}\cot\left(\dfrac{\pi}{7}p - \dfrac{11\pi}{13}\right)$

NOTE: Problems like (P7) & (P8) are not in the textbook [1], therefore such problems will not be on an exam. Problem (P6) is complex enough algebraically that a similar problem may be a possible bonus question on an exam.

References

[1] M. L. Lial, J. E. Hornsby, D. Schneider. *Trigonometry.* Pearson, Boston, MA, 9th Edition, 2009.