Texas Tech University
MATH1321 -- Trigonometry

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List of Identities to Study for Final Exam

$\sin\theta \equiv \frac{y}{r},\ \cos\theta \equiv \frac{x}{r},\ \tan\theta \equiv \frac{y}{x},\ \csc\theta \equiv \frac{r}{y},\ \sec\theta \equiv \frac{r}{x},\ \cot\theta \equiv \frac{x}{y}$ (Point-wise Trig Definitions)

REMEMBER: $r^2 = x^2 + y^2$ and $r > 0$ since r is the distance from the origin to the point (x, y)

$\sin\theta \equiv \frac{OPP}{HYP},\ \cos\theta \equiv \frac{ADJ}{HYP},\ \tan\theta \equiv \frac{OPP}{ADJ}$
$\csc\theta \equiv \frac{HYP}{OPP},\ \sec\theta \equiv \frac{HYP}{ADJ},\ \cot\theta \equiv \frac{ADJ}{OPP}$ (Right-Triangle Trig Definitions [Acute-Angle Definitions])

Useful mnemonic device: SOH-CAH-TOA

$\csc\theta = \frac{1}{\sin\theta},\ \sec\theta = \frac{1}{\cos\theta},\ \cot\theta = \frac{1}{\tan\theta}$ (Reciprocal Identities)

$\tan\theta = \frac{\sin\theta}{\cos\theta},\ \cot\theta = \frac{\cos\theta}{\sin\theta}$ (Quotient Identities)

$\sin^2\theta + \cos^2\theta = 1,\ \tan^2\theta + 1 = \sec^2\theta,\ 1 + \cot^2\theta = \csc^2\theta$ (Pythagorean Identities)

$\sin(-\theta) = -\sin\theta,\ \cos(-\theta) = \cos\theta,\ \tan(-\theta) = -\tan\theta$
$\csc(-\theta) = -\csc\theta,\ \sec(-\theta) = \sec\theta,\ \cot(-\theta) = -\cot\theta$ (Negative-Angle Identities)

$\sin\theta = \cos(90^\circ - \theta),\ \cos\theta = \sin(90^\circ - \theta)$
$\sec\theta = \csc(90^\circ - \theta),\ \csc\theta = \sec(90^\circ - \theta)$ (Co-function Identities)
$\tan\theta = \cot(90^\circ - \theta),\ \cot\theta = \tan(90^\circ - \theta)$

$\cos(A + B) = \cos A\cos B - \sin A\sin B,\ \cos(A - B) = \cos A\cos B + \sin A\sin B$
$\sin(A + B) = \sin A\cos B + \cos A\sin B,\ \sin(A - B) = \sin A\cos B - \cos A\sin B$
$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B},\ \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$ (Sum/Difference Identities)

$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$
$\sin 2\theta = 2\sin\theta\cos\theta$
$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$ (Double-Angle Identities)

$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1 + \cos\theta}{2}}$
$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1 - \cos\theta}{2}}$ (Half-Angle Identities)
$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1 - \cos\theta}{1 + \cos\theta}} = \frac{\sin\theta}{1 + \cos\theta} = \frac{1 - \cos\theta}{\sin\theta}$

REMARK: The choice of 'plus' or 'minus' preceding the square-root depends on which quadrant $\frac{\theta}{2}$ resides.

REMARK: In general, the 1st form of the half-angle tangent identity (involving only cosine) is preferred.

NOTE: The Product-to-Sum & Sum-to-Product Identities (Section 5.5, pgs 237-238) will **NOT** be on the Final Exam!