

PRACTICE EXAM 1 – Limits and Derivatives (Sections 9.1 - 9.8)

- * Print your name at the upper-right corner of exam (and your initials on every page you use)
- * Write your work and answers on paper provided – leave at least a half-inch margin on right-side of each page.
- * Be sure to show appropriate, sufficient work – merely asserting a calculator result is not enough!

PROBLEMS:

1. a) Evaluate the following limit either using a table & calculator or analytically: $\lim_{x \rightarrow -1} \frac{10x^4 - 10}{x^8 - 1}$
- b) Evaluate the following limit **analytically** (i.e. no table, no calculator): $\lim_{x \rightarrow \infty} \frac{5x^4 + x^2 - 3x + 2}{3x^4 + 2x^3 - x - 1}$
2. Given $s(t) = \begin{cases} 1 - t^3 & , t < 2 \\ t^2 - 11 & , 2 \leq t < 9 \\ 3 & , t = 9 \\ 2t - \sqrt{t} & , t > 9 \end{cases}$,
- a) Evaluate the one-sided limit: $\lim_{t \rightarrow 2^-} s(t)$
- b) Evaluate the one-sided limit: $\lim_{t \rightarrow 2^+} s(t)$
- c) Evaluate the two-sided limit: $\lim_{t \rightarrow 2} s(t)$
- d) Is $s(t)$ continuous at $t = 2$? e) Is $s(t)$ continuous at $t = 5$? f) Is $s(t)$ continuous at $t = 9$? (Justify answers)
3. a) Given $f(x) = 2x - 5$, find $f'(x)$ using the definition of the derivative.
- b) Given $g(x) = 4x^2 + x$, find $g'(x)$ using the definition of the derivative.
4. a) Given $h(x) = 30x^{2/3} - \frac{8}{x^{50}} + \frac{1}{\sqrt[5]{x^7}}$, find $h'(-1)$.
- b) Given $y = 4x^{1000} + 10x^3 - 2x^2 + 100\sqrt{x} - 10^9$, find y' .
5. a) Given $f(x) = (3x^2 - x - 2)(x^{10} - x^6 - 1)$, find the slope of the tangent line to $f(x)$ at point $(1, 0)$.
- b) Given $g(x) = \frac{3x + 1}{x^9 - 4x^5}$, find the equation of the tangent line to $g(x)$ at $x = -1$.
6. a) Given $w(t) = \frac{1}{2}(2t - 1)^{10}$, find $\frac{d^2w}{dt^2}$, b) If $v = 3u^4$, and $u = 2x + 1$, find $\frac{dv}{dx}$.
7. Given $f(x) = -5e^{-4x}$, a) find $f'(x)$. b) is f increasing, decreasing, or constant at $x = 0$? (Justify answer)
8. Given $g(z) = z^6 \ln z$, a) find $g'(z)$. b) is g differentiable at $z = 0$? (Justify answer)
9. The demand for a store's cordless phones obeys the relationship $p = -0.02x + 800$ ($0 \leq x \leq 20,000$) where p denotes the unit price of the phone (in dollars) and x denotes the quantity demanded.
- a) Find the revenue function $R(x)$. b) Find the marginal revenue function $R'(x)$.
10. The total weekly cost in dollars in producing x DVD's is given by $C(x) = 2500 + 2.2x$ ($0 \leq x \leq 7000$)
- a) What is the marginal cost when $x = 1000$? b) Find the average cost function $\bar{C}(x)$.

BONUS QUESTIONS: Expect 2-4 bonus questions either extending any of the above problems or testing knowledge/interpretation of limits, continuity, discontinuity, differentiability, and/or derivatives.

Example 1: In Problem 6(b) above, a) find $\frac{d^2v}{dx^2}$ b) find $\frac{d^3v}{dx^3}$ c) find $\frac{d^{(20)}v}{dx^{(20)}}$

Example 2: In Problem 7 above, a) find $f''(x)$ b) find $f'''(x)$ c) find $f^{(4)}(x)$ d) find $f^{(n)}(x)$