TTU - MATH1331 (Business Calculus) - JOSH ENGWER - 9/18/2011

PRACTICE EXAM 1 - SOLUTIONS (Sections 9.1 - 9.8)

* For most problems, only the answer and maybe a hint is given – you have to fill in the details!

* Exceptions to the above : Problems 2(c)(d)(e)(f), 3(a)(b), 6(b), 7(b), 8(b), 9(a), 10(b)

1. a) $\lim_{x \to -1} \dfrac{10x^4 - 10}{x^8 - 1} = \boxed{5}$ (Substitution of $x = -1$ yields $\frac{0}{0}$, which signals to simplify expression)

   b) $\lim_{x \to \infty} \dfrac{5x^4 + x^2 - 3x + 2}{3x^4 + 2x^3 - x - 1} = \boxed{\dfrac{5}{3}}$ (Divide top & bottom of fraction by x^4, then take limits)

2. a) $\lim_{t \to 2^-} s(t) = \boxed{-7}$

   b) $\lim_{t \to 2^+} s(t) = \boxed{-7}$

   c) $\lim_{t \to 2} s(t) = \boxed{-7}$ (since $\lim_{t \to 2^-} s(t) = \lim_{t \to 2^+} s(t) = -7$)

   d) $s(t)$ is $\boxed{\text{continuous}}$ at $t = 2$ since $s(2)$ exists, $\lim_{t \to 2} s(t)$ exists, and $\lim_{t \to 2} s(t) = s(2)$

   e) $s(t)$ is $\boxed{\text{continuous}}$ at $t = 5$ since $t = 5$ is <u>inside</u> the interval $[2, 9)$, not on the boundary, and the piece for that interval is $t^2 - 11$, which is a polynomial and polynomials are continuous everywhere

   f) $s(t)$ is $\boxed{\text{NOT continuous}}$ at $t = 9$ since $\lim_{t \to 9^-} s(t) = 70 \neq 15 = \lim_{t \to 9^+} s(t) \Rightarrow \lim_{t \to 9} s(t)$ DNE

3. a) Using the <u>definition</u> of the derivative of $f(x) = 2x - 5$:

   $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{[2(x+h) - 5] - [2x - 5]}{h} = \lim_{h \to 0} \frac{2x + 2h - 5 - 2x + 5}{h}$$
   $$= \lim_{h \to 0} \frac{2h}{h} = \lim_{h \to 0} 2 = \boxed{2}$$

   b) Using the <u>definition</u> of the derivative of $g(x) = 4x^2 + x$:

   $$g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h} = \lim_{h \to 0} \frac{[4(x+h)^2 + (x+h)] - [4x^2 + x]}{h}$$
   $$= \lim_{h \to 0} \frac{[(4x^2 + 8hx + 4h^2) + (x+h)] - [4x^2 + x]}{h} = \lim_{h \to 0} \frac{h[(8x+1) + h]}{h} = \lim_{h \to 0} [(8x+1) + h]$$
   $$= (8x + 1) + 0 = \boxed{8x + 1}$$

4. a) $h'(-1) = \boxed{-\dfrac{2107}{5}}$ b) $y' = \boxed{4000x^{999} + 30x^2 - 4x + 50x^{-1/2}}$

5. a) Slope of tangent line to $f(x)$ at point $(1, 0) = f'(1) = \boxed{-5}$

   b) Slope of tangent line is $m = g'(-1) = -\dfrac{13}{9}$, point $(x_0, y_0) = (-1, g(-1)) = \left(-1, -\dfrac{2}{3}\right)$.

   Thus, equation of line is $\boxed{y = -\dfrac{13}{9}x - \dfrac{19}{9}}$

6. a) $\dfrac{d^2w}{dt^2} = \boxed{180(2t-1)^8}$

   b) Using Chain Rule (Leibniz form), $\dfrac{dv}{dx} = \dfrac{dv}{du}\dfrac{du}{dx} = (12u^3)(2) = 24u^3 = \boxed{24(2x+1)^3}$

   (Remember, $\dfrac{dv}{dx}$ means the derivative of v must be written **in terms** of x, not u.)

7. a) $f'(x) = \boxed{20e^{-4x}}$ b) f is $\boxed{\text{increasing}}$ at $x = 0$ because $f'(0) = 20e^{-4(0)} = 20(1) = 20 > 0$

8. a) $g'(z) = \boxed{6z^5 \ln z + z^5}$ b) g is $\boxed{\text{NOT differentiable}}$ at $z = 0$ because $g'(0)$ DNE

9. a) Revenue function $R(x) = xp = x(-0.02x + 800) \Rightarrow \boxed{R(x) = -0.02x^2 + 800x}$

   b) Marginal revenue function $\boxed{R'(x) = -0.04x + 800}$

10. a) Marginal cost when $x = 1000 = C'(1000) = \boxed{\$2.20/\text{week}}$

    b) Average cost function $\overline{C}(x) = \dfrac{C(x)}{x} \Rightarrow \boxed{\overline{C}(x) = 2.2 + \dfrac{2500}{x}}$

<u>BONUS QUESTIONS:</u>

Example 1: a) $\dfrac{d^2v}{dx^2} = \boxed{144(2x+1)^2}$ b) $\dfrac{d^3v}{dx^3} = \boxed{576(2x+1)}$

c) $\dfrac{d^{(20)}v}{dx^{(20)}} =$??? (come by my office hours and tell me what your answer is)

Example 2: a) $f''(x) = \boxed{-80e^{-4x}}$ b) $f'''(x) = \boxed{320e^{-4x}}$ c) $f^{(4)}(x) = \boxed{-1280e^{-4x}}$

d) $f^{(n)}(x) =$??? (come by my office hours and tell me what your answer is)