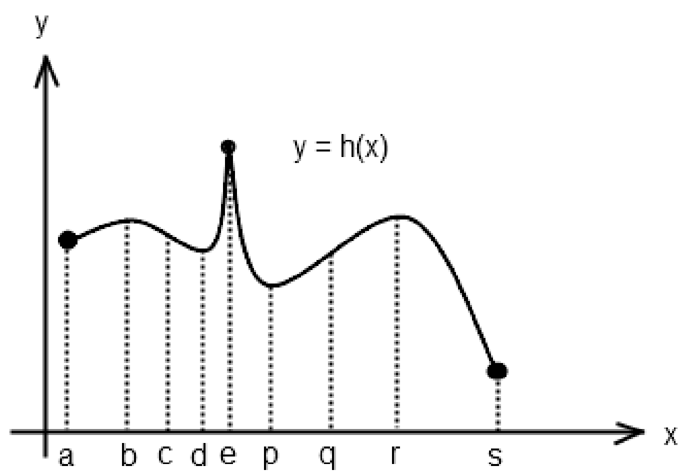
TTU - MATH1331 (Business Calculus) - JOSH ENGWER 4/4/2012

Practice Exam 3 – Applications of the Derivative (Sections 10.1,10.2,10.4,10.5) & Regression

* Print your name at the upper-right corner (and your initials on every page you use)
* Write your work and answers on paper provided – leave at least a half-inch margin on right-side of each page
* Be sure to show appropriate, sufficient work – merely asserting a calculator result is not enough!

PROBLEMS:

1. Given the following graph of $h(x)$, find:
   a) the intervals where $h(x)$ is increasing
   b) the intervals where $h(x)$ is decreasing
   c) the relative extrema (minima & maxima) of $h(x)$
   d) the intervals where $h(x)$ is concave up
   e) the intervals where $h(x)$ is concave down
   f) the inflection points, if any, of $h(x)$
   g) the absolute minimum of $h(x)$
   h) the absolute maximum of $h(x)$



2012/graph1.png

2. Given $f(x) = x - \dfrac{1}{x}$, find:
   a) the intervals where $f(x)$ is increasing
   b) the intervals where $f(x)$ is decreasing
   c) the relative extrema (minima & maxima), if any, of $f(x)$
   d) the intervals where $f(x)$ is concave up
   e) the intervals where $f(x)$ is concave down
   f) the inflection points, if any, of $f(x)$

   (HINT: find $f'(x)$, $f''(x)$, build a table, and interpret the results)

3. Given $g(x) = 2x^3 - 6x^2 + 6x + 1$, find:
   a) the intervals where $g(x)$ is increasing
   b) the intervals where $g(x)$ is decreasing
   c) the relative extrema (minima & maxima), if any, of $g(x)$
   d) the intervals where $g(x)$ is concave up
   e) the intervals where $g(x)$ is concave down
   f) the inflection points, if any, of $g(x)$
   g) the absolute extrema (minimum & maximum) of $g(x)$ over interval $[-3, 3]$

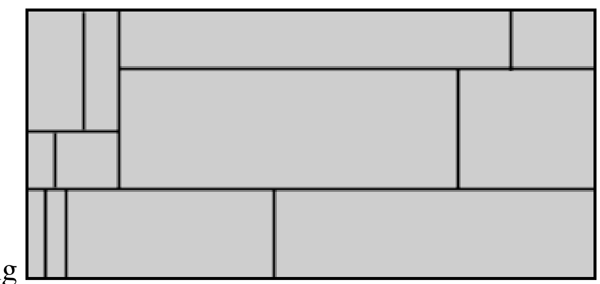
4. Odyssey Travel Agency's monthly profit (in thousands of dollars) depends on the amount of money x (in thousands of dollars) spent on advertising each month according to

$$P(x) = -x^2 + 8x + 20 \qquad (0 \leq x \leq 10)$$

a) Find $P'(x)$.
b) To maximize its monthly profits, what should be the monthly advertising budget? (Build a table)

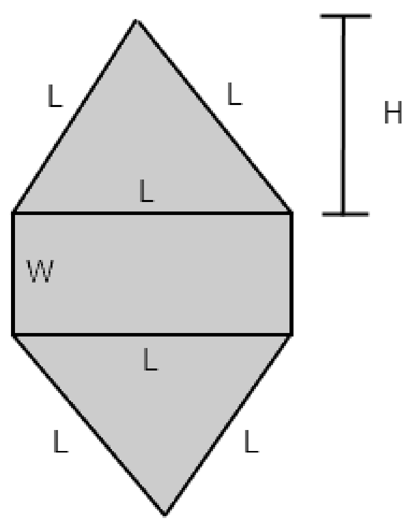
5. Mitch wishes to have a rectangular-shaped garden in his backyard.
He has 100 ft of fencing with which to enclose his garden in the following configuration. (see below)
(The gray shading represents the garden. Also, the "thickness" of the fencing is negligible.)
Find the dimensions for the largest garden he can have if he uses all of the fencing.

(NOTE: Area of a rectangle = Length $\times$ Width)

2012/garden1.png 

6. Now suppose Mitch wants a garden of the following shape and fence configuration. (see below)
He wants the area of the garden to be 100 ft^2.
(The gray shading represents the garden. Also, the "thickness" of the fencing is negligible.)
Find the dimensions for the garden such that the least amount of fencing is used.

(NOTE: Area of a triangle = $\frac{1}{2}$ Base $\times$ Height)
(NOTE: Height of each equilateral triangle shown is $H = \frac{\sqrt{3}}{2}L$)



2012/garden2.png

7. Given the following set of data points:

x	-1	0	1	2	3	4	5
y	4	1	2	0.1	1	1	2

a) Find the best-fitting linear model: $y = ax + b$ b) Compute the correlation coefficient R^2

c) Is the correlation positive, negative, or neither? (Hint: Look at the slope of the best-fit linear model)

Use calculator's linear regression function.
Only use the following formulas if your calculator cannot compute linear regression directly:

$$a = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}, \qquad b = \frac{(\sum y)(\sum x^2) - (\sum x)(\sum xy)}{n(\sum x^2) - (\sum x)^2},$$

$$R^2 = \frac{\left[n(\sum xy) - (\sum x)(\sum y)\right]^2}{\left[n(\sum x^2) - (\sum x)^2\right]\left[n(\sum y^2) - (\sum y)^2\right]}$$

NOTE: n is the number of data points.

BONUS QUESTIONS:

Questions (B1), (B2), (B3) all reference Problem 7 above.

(B1) Find the best-fitting cubic model: $y = ax^3 + bx^2 + cx + d$ (Use calculator)

(B2) Find the best-fitting exponential model: $y = ab^x$ (Use calculator)

(B3) Which of the above three models is the best-fit (most accurate) overall? (Justify your answer)

(B4) Referring to Problem 3, find the absolute extrema of $g(x)$ over interval $[0, \infty)$