TTU - MATH1331 (Business Calculus) - JOSH ENGWER - 11/17/2011

**PRACTICE EXAM 3 – Anti-Derivatives & Integration (Chapter 11)**

* Be sure to **show appropriate, sufficient work** – merely asserting a calculator result is not enough!

PROBLEMS:

1. Find each indefinite integral: a) $\int \left(x^4 - 2x^{7/3} + \frac{4}{x^2} - \sqrt{x}\right) dx$ b) $\int \frac{(\ln z)^5}{z}\, dz$ c) $\int (w^2+1)(1-w^3)\, dw$

2. Evaluate each integral: a) $\int_{-1}^{2} \frac{3x}{x^2+4}\, dx$ b) $\int_{1}^{4} \left(e^t + \frac{1}{\sqrt[3]{t^2}} - t^{-9/2}\right) dt$ c) $\int_{1}^{2} \sqrt{2v}(v - 4\sqrt{3v} - 1)\, dv$

3. Find the average value of the function $h(x) = \dfrac{x}{\sqrt{x^2+16}}$ over the interval $[0, 3]$.

4. a) Find the area of the region under the curve $f(x) = 3x^2 + 2x + 1$ from $x = -1$ to $x = 2$.
b) Find the area between the curves $g(t) = 10 + e^{2t}$ and $h(t) = 4 + \ln t$ from $t = 1$ to $t = 3$.
[HINT: $g(t)$ and $h(t)$ do NOT intersect in the prescribed interval. Also $\int \ln x\, dx = x \ln x - x + C$]

5. Find the area of the region that is completely enclosed by the curves $f(x) = x^4$ and $g(x) = x$.
[HINT: Find the two points of intersection and determine which curve is larger in the interval in between]

6. The quantity demanded $x$ (in hundreds) of Sportsman tents per week is given by: $D(x) = -0.1x^2 - x + 40$
The supply function is $S(x) = 0.1x^2 + 2x + 20$, where $x$ is in hundreds of tents.

   If the market price is set at the market equilibrium price,
   find the consumers' surplus ($CS$) and the producers' surplus ($PS$).
   [HINT: The **market equilibrium** is the point $(\bar{x}, \bar{p})$ where $D(x)$ and $S(x)$ intersect.]

7. Chi-Tai plans to deposit \$4000/year in his Keogh Retirement Account.
   If interest is compounded continuously at the rate of 8%/year,
   how much will he have in his retirement account after 20 years?
   [HINT: Regular deposits to a retirement account is an **annuity**]

8. Alicia purchased a 10-year franchise for a health spa that's expected to generate income at \$80,000/year.
   If the prevailing interest rate is 10%/year compounded continuously, find the present value of the franchise.
   [HINT: This is an **income stream**]

9. A certain country's income distribution is described by the Lorenz curve:

   $$L(x) = \frac{17}{18}x^2 + \frac{1}{18}x$$

   a) Compute $L(0.3)$ and $L(0.6)$
   b) Interpret your results from part (a)
   c) Compute the coefficient of inequality (**Gini index**) for $L(x)$

BONUS QUESTIONS:

(B1) Determine whether each function is even, odd, or neither (justify answers):
a) $f(x) = 2x^{59} - \frac{1}{7}x^{11} - x$ b) $g(z) = \dfrac{1 - z^{10}}{2 - z^6 + z^{100}}$ c) $h(w) = e^w$ d) $\varphi(t) = 7 - t + t^{19}$

(B2) Find the area of region bounded by the curve $f(x) = x^3 - 8$, the x-axis, and vertical lines $x = -4$ & $x = 5$.
[HINT: Treat this as an "area between two curves" problem with the second curve $g(x) =$ ???]
[HINT 2: Find the one point of intersection of the curves $f(x)$ and $g(x)$]

[Possible bonus questions include properties of anti-derivatives, integrals, and Lorenz curves]