APPLICATIONS OF THE DEFINITE INTEGRAL

CONSUMERS' SURPLUS:

$CS \equiv$ **consumers' surplus** is the **difference** between
what consumers would be willing to pay for $\bar{x}$ units of a commodity and
what they actually pay for them.

$$CS = \int_0^{\bar{x}} D(x)\,dx - \bar{p}\bar{x} = \int_0^{\bar{x}} [D(x) - \bar{p}]\,dx$$

$D(x)$ is the **demand function** that relates the unit price p of a commodity to the quantity x demanded of it.
$\bar{p}$ is the **fixed unit market price**. $\bar{x}$ is the **quantity demanded** in the market.

PRODUCERS' SURPLUS:

$PS \equiv$ **producers' surplus** is the **difference** between
what the suppliers actually receive and what they would be willing to receive.

$$PS = \bar{p}\bar{x} - \int_0^{\bar{x}} S(x)\,dx = \int_0^{\bar{x}} [\bar{p} - S(x)]\,dx$$

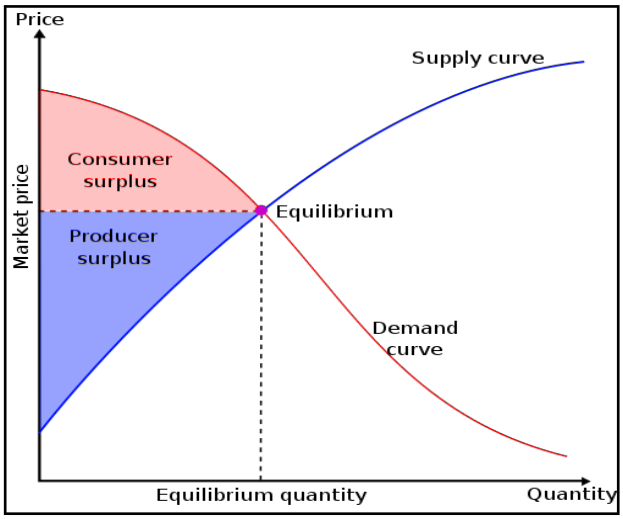
$S(x)$ is the **supply function** that relates the unit price p of a commodity to the quantity x that the supplier sells.
$\bar{p}$ is the **fixed unit market price**. $\bar{x}$ is the **quantity supplied** to the market.

REMARK: The point where the demand & supply curves intersect is called **market equilibrium**.

TOTAL FUTURE VALUE OF AN INCOME STREAM:

$$FV = e^{rT} \int_0^T R(t)e^{-rt}\,dt$$

$R(t)$ is the **rate of income generation** at time t in dollars/year.
r is the **interest rate** compounded continuously.
T is the **term** in years.



PRESENT VALUE OF AN INCOME STREAM:

$$PV = \int_0^T R(t)e^{-rt}\,dt$$

AMOUNT OF AN ANNUITY:

An **annuity** is a **sequence of payments or deposits made at regular time intervals**
(e.g. monthly insurance payments, monthly mortgage, regular IRA deposits, regular savings deposits, ...)

$$A = \frac{mP}{r}(e^{rT} - 1)$$

P is the **size of each payment/deposit** in the annuity.
r is the **interest rate** compounded continuously.
T is the **term** of the annuity in years.
m is the **number of payments/deposits per year**.

PRESENT VALUE OF AN ANNUITY:

$$PV = \frac{mP}{r}(1 - e^{-rT})$$

References

[1] S. Tan, *Applied Mathematics for the Managerial, Life, and Social Sciences*. Brooks Cole, Belmont, CA, 5th Edition, 2008.

[2] Economic Surplus image from Wikipedia.
`http://en.wikipedia.org/wiki/File:Economic-surpluses.svg`