SUMMARY OF THE DEFINITE INTEGRAL

NOTATION FOR THE DEFINITE INTEGRAL:

$\int_a^b f(x)\,dx$ reads "The integral of $f(x)$ from a to b with respect to x"

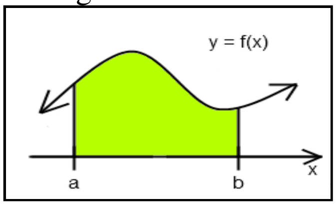
a and b are called the **limits of integration**, and $a < b$.

Here, x is called a **dummy variable** – any variable can be used since it just acts as a **label**:

For example, $\int_a^b x^2\,dx = \int_a^b y^2\,dy = \int_a^b z^2\,dz = \int_a^b w^2\,dw = \cdots$

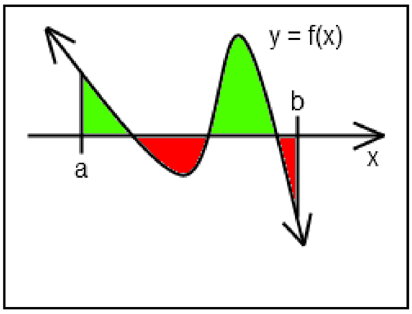
DEFINITION OF THE DEFINITE INTEGRAL:

$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} hf(x_i)$, where $h = \frac{b-a}{n}$ is the **step size**, and n is the # of rectangles used.
(i.e. approximating the area under the curve improves as more rectangles are used)



INTERPRETATION OF THE DEFINITE INTEGRAL:

Geometrically, $\int_a^b f(x)\,dx$ is the area bounded by curve $f(x)$, the x-axis & vertical lines $x = a$ & $x = b$.



Note that any area **below the x-axis** is considered **negative** area.

In applications, if $f'(x)$ is the **rate of change** of a quantity,
then $\int_a^b f'(x)\,dx$ is the **total amount** of that quantity from $x = a$ to $x = b$.

Examples:

If $s'(t)$ is the **speed** at time t, then $\int_{t_0}^{t_1} s'(t)\,dt$ is the **total distance** traveled from time t_0 to time t_1.

If $C'(x)$ is the **marginal cost** of making the x^{th} widget,
then $\int_0^8 C'(x)\,dx$ is the **total cost** of making the first 8 widgets,
and $\int_5^{20} C'(x)\,dx$ is the **total cost** of making the 6^{th} through 20^{th} widgets.

DEFINITE INTEGRAL RULES:

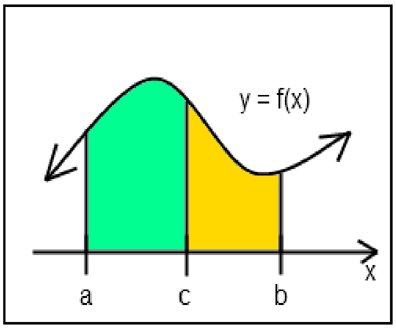
Constant Multiple Rule: $\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$ [k is a real number]

Sum/Difference Rule: $\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

Point Rule: $\int_a^a f(x)\,dx = 0$

Flip Interval Rule: $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

Lump Intervals Rule: $\int_a^c f(x)\,dx + \int_c^b f(x)\,dx = \int_a^b f(x)\,dx$ [$a < c < b$]



References

[1] S. Tan, *Applied Mathematics for the Managerial, Life, and Social Sciences*. Brooks Cole, Belmont, CA, 5th Edition, 2008.